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On the coexistence of cooperators, defectors and conditional cooperators in the multiplayer iterated Prisoner's Dilemma

Jelena Grujić [a], José A. Cuesta [a,*], Angel Sánchez [a,b]

[a] Grupo Interdisciplinar de Sistemas Complejos (GISC), Departamento de Matemáticas, Universidad Carlos III de Madrid, Avenida de la Universidad 30, 28911 Leganés, Madrid, Spain
[b] Instituto de Biocomputación y Física de Sistemas Complejos (BIFI), Universidad de Zaragoza, Campus Río Ebro, 50018 Zaragoza, Spain

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ABSTRACT

Recent experimental evidence [Grujić Fosco, Araujo, Cuesta, Sánchez, 2010. Social experiments in the mesoscale: humans playing a spatial Prisoner's dilemma. PLoS ONE 5, e13749] on the spatial Prisoner's Dilemma suggests that players choosing to cooperate or not on the basis of their previous action and the actions of their neighbors coexist with steady defectors and cooperators. We here study the coexistence of these three strategies in the multiplayer iterated Prisoner's Dilemma by means of the replicator dynamics. We consider groups with $n=2$, 3, 4 and 5 players and compute the payoffs to every type of player as the limit of a Markov chain where the transition probabilities between actions are found from the corresponding strategies. We show that for group sizes up to $n=4$ there exists an interior point in which the three strategies coexist, the corresponding basin of attraction decreasing with increasing number of players, whereas we have not been able to locate such a point for $n=5$. We analytically show that in the limit $n\to\infty$ no interior points can arise. We conclude by discussing the implications of this theoretical approach on the behavior observed in experiments.

1. Introduction

In the past few years, different mechanisms have been proposed to explain the origin and stability of cooperation (Nowak, 2006). One of these mechanisms involves assortment of cooperators (Fletcher and Doebeli, 2009), in particular through the existence of a spatial or social structure dictating who interacts with whom (cf. network reciprocity in Nowak, 2006). Cooperators might then interact mainly with each other and keep the benefits of cooperation to the extent that they perform better than defectors or free riders in peripheral positions. This idea stems from the work by Nowak and May (1992b), who carried out a simulation of the iterated Prisoner's Dilemma (PD) (Rapoport and Guyer, 1966; Axelrod and Hamilton, 1981) on a lattice in which every individual interacted with her eight nearest neighbors. Their finding of sizable proportions of cooperative actions even when the temptation to defect was quite large stimulated a large amount of work on evolutionary game theory on graphs (for reviews see, e.g., Szabó and Fáth, 2007; Roca et al., 2009). Unfortunately, in spite of the large body of theoretical work devoted to this issue, it has not been possible to reach a general conclusion about how the existence of structure on a population could promote cooperation: indeed, it was shown that the emergence and survival of cooperative behaviors depended so crucially on the details of the models that their applicability to real life situations was dubious, at best.

In view of this situation, in the last few years a number of groups have carried out experiments to probe the relationship between population structure and cooperation with real human subjects (Kirchkamp and Nagel, 2007; Traulsen et al., 2010; Grujić et al., 2010). Arguably, the main conclusion of this research is that lattice-like structures do not seem to promote cooperation, at least not to an extent different from what is found in dyadic or small group experiments (Kagel and Roth, 1995; Camerer, 2003). While the lack of promotion of cooperation is well established, the reasons proposed by the different teams to explain the experimental observations are different, and there is no consensus yet as to what is the way the subjects updated their decisions during the experiment. In particular, Kirchkamp and Nagel (2007) focused on disproving the imitation strategy proposed by Nowak and May (1992b), a conclusion also supported by Grujić et al. (2010). On the other hand, Traulsen et al. (2010) fitted their results to a payoff-dependent imitation behavior—Fermi rule (Szabó and Töke, 1998)—finding that they needed a large amount of random mutation to explain their observations.

In the above context, the analysis carried out by Grujić et al. (2010) brought in an alternative way to understand the experimental observations by building upon the idea of reciprocity

* Corresponding author.
E-mail addresses: jgrujic@math.uc3m.es (J. Grujić), cuesta@math.uc3m.es (J.A. Cuesta), anxo@math.uc3m.es (A. Sánchez).
URLS: http://gisc.uc3m.es/~cuesta (J.A. Cuesta), http://allariz.uc3m.es/~anxosanchez/ (A. Sánchez).

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(Trivers, 1971), i.e., the fact that individuals behave depending on the actions of their partners in the past. In iterated two-player games, this idea has been studied through the concept of reactive strategies (Nowak and Sigmund, 1989a,b, 1990; Nowak and May, 1992a) (see Sigmund, 2010 for a comprehensive summary on this matter), the most famous of which is Tit-For-Tat (Axelrod and Hamilton, 1981), given by playing what the opponent played in the previous run. Reactive strategies generalize this idea by considering that players choose their action among the available ones with probabilities that depend on the previous action of the opponent. For the simple case of two strategies (say C and D), players choose C with probability p following a C from their partner and with probability q after a D from their partner. Subsequently this idea was further developed by considering memory-one reactive strategies (Nowak et al., 1995; Sigmund, 2010), in which the probabilities depend on the previous action of both the focal player and her opponent—i.e., the focal player would choose C with some probability following a (C,C) outcome, and so on.

In iterated multiplayer games, such as public goods games or multiplayer Prisoner's Dilemmas (IMPD), reciprocity arises in the form of conditional cooperation (Fischbacher et al., 2001; Gächter, 2007): individuals are willing to contribute more to a public good the more others contribute. Conditional cooperation has been observed a number of times in public goods experiments (Croson, 2007; Fischbacher and Gächter, 2010), often along with a large percentage of free-riders. The experiment by Traulsen et al. (2010) showed also evidence for such a behavior in a spatial setup. Grujić et al. (2010) extended this idea in their analysis to include the dependence of the focal player's previous action, introducing the so-called moody conditional cooperation (cf. Fig. 1). In this strategy, players are more prone to cooperate after having cooperated than after having defected, and in the first case they are more cooperative the more cooperative neighbors they have. This behavior has not been reported before in spatial games and appears to be a natural extension of the reactive strategy idea to multiplayer games (among the very many other extensions one can conceive). On the other hand, and from an economic viewpoint, which is an important part of the analysis of human behavior, this type of strategy update scheme responds to the often raised questions on payoff-based rules. In economic interactions it is usually the case that agents perceive the others' actions but not how much do they benefit from them, and therefore the use of action updates depending, e.g., on the payoff differences, may be questionable. This seems to be the case even if this information is explicitly supplied to the players (Grujić et al., 2010).

Interestingly, the conclusion of Grujić et al. (2010) had a new feature as compared to the other two experiments (Kirchkamp and Nagel, 2007; Traulsen et al., 2010), namely the heterogeneity of the population: aside from the already mentioned moody conditional cooperators, there were a large minority of defectors, i.e., players that defected all or almost all the time, and a few cooperators, that cooperated at practically all rounds. This heterogeneity, also found to be very important in public goods experiments (Fischbacher and Gächter, 2010) had also been observed in four-player experiments by Kurzban and Houser (2005), who reported that their subjects could be roughly classified into three main types, including defectors, cooperators and conditional cooperators (called reciprocators in the original work), albeit they did not check for dependences on the past actions of the players either. Both Kurzban and Houser (2005) and Grujić et al. (2010) checked that the payoffs obtained by every type of player were more or less the same, thus suggesting that the population in the lattice experiment might be at an evolutionary equilibrium.

In this paper we address the question of the existence and stability of such a heterogeneous or mixed equilibrium in the multiplayer iterated Prisoner's Dilemma. It is important to understand that we are not addressing the issue of the evolutionary explanation of moody conditional cooperation. This is a very interesting but also very difficult task, and in fact we do not even have an intuition as to how one can address this problem in a tractable manner. Our goal is then to understand whether or not the coexistence of moody conditional cooperators, defectors, and a small percentage of cooperators, as observed in the experiment, is theoretically possible. In so doing, we will shed light on experimental and theoretical issues at the same time. On the experimental side, our results show that there is coexistence for groups of two or three players for parameters reasonably close to those found in the experiment, but not for larger groups. As we will see in the Discussion section, this prediction has important consequences related to the adequacy of replicator dynamics to describe the experimental result or to the cognitive capabilities of human subjects in dealing with large groups. We will also discuss there the ways in which our theoretical approach and the experiment may differ, something that can also have implications of its own. On the theoretical side, we present an analysis of a population of players interacting through a multiplayer Prisoner's Dilemma including strategies that generalize the ideas behind reactive strategies, as mentioned above. To our knowledge, this

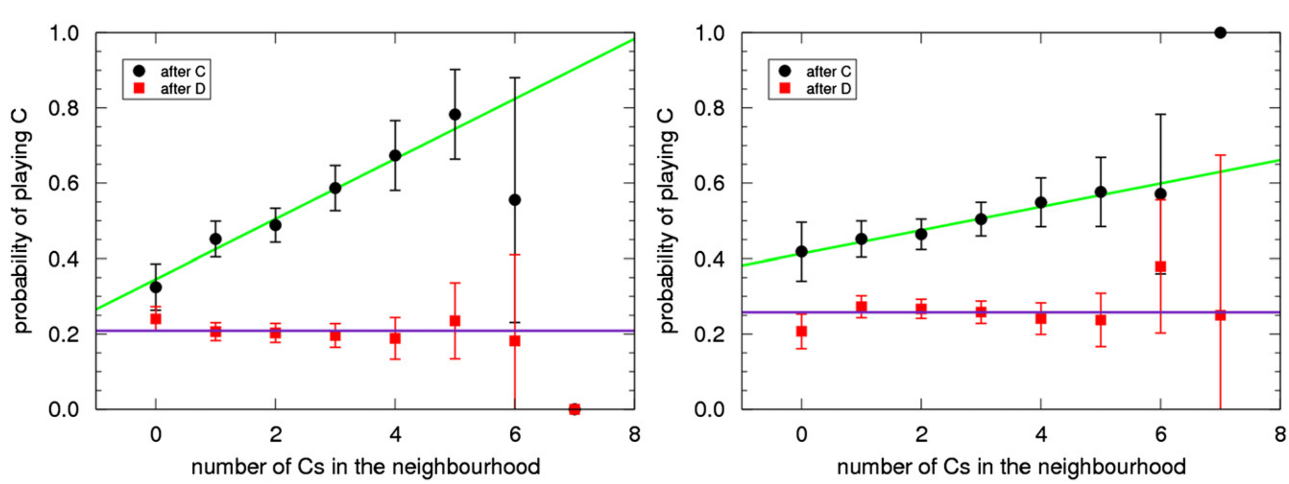

Fig. 1. Probabilities of cooperating after playing C or D, conditioned to the context (number of cooperators in the previous round) in the two experiments by Grujić et al. (2010). Parameters of the fitted lines will be used later as inputs for our replicator dynamics study. The line fitted to the probabilities of cooperation after playing D is strictly horizontal.