Journal of Theoretical Biology 291 (2011) 29–32

Contents lists available at SciVerse ScienceDirect

Journal of Theoretical Biology

journal homepage: www.elsevier.com/locate/yjtbi

# A mathematical model for the Andean Tiwanaku civilization collapse: Climate variations

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ARTICLE INFO

*Article history:*
Received 18 May 2011
Received in revised form
24 August 2011
Accepted 7 September 2011
Available online 21 September 2011

*Keywords:*
Mathematical modelling
Nonlinear systems

ABSTRACT

We propose a mathematical nonlinear model for the Tiwanaku civilization collapse based on the assumption, supported by archeological data, that a drought caused a lack of the main resource, water. We evaluate the parameter of our model using archaeological data. According to our numerical simulation the population core should have decreased from 45 000 to 2000 inhabitants due to lake surface contraction.



## 1. Introduction: the Tiwanaku civilization

Why do some societies collapse? The answer to this is varied and complex. Tainter (2007) points this out in his book. There are two major opposing responses: on one side is the belief in internal causes, that social conflict destroys the established order; on the other side is the concept of an uncontrollable external input, such as severe climatological change. In a realistic case it is reasonable to assume that both aspects are present. Tainter's book points out that in principle, a complex society could manage any external input, or stated otherwise, a complex society could eventually adapt to the external change. The collapse, if it occurs, has an internal dynamic due to poor management strategies. Diamond (2005) presents a different concept. According to him, among several causes, one could be that an external input could directly produce a collapse. We propose that the latter concept, rather than the former, played a more important role in the collapse of a pre-Columbian Andean civilization, the Tiwanaku. We stress the fact that in literature are present rather sophisticated modelling approaches related to the collapse of Tiwanaku as studied by Griffin and Stanish (2007). In that work, an agent-based model is considered and an interdisciplinary aspect is studied. In our study we limit ourselves to a macro-dynamic based on exploitation of the natural resources.

The Tiwanaku civilization started around 300 B.C. reaching a stable number of inhabitants around 1000 A.D. The Tiwanaku settlement was situated near the southern part of the Titikaka Lake which was known either as Minor Lake or Wiñaymarka lake (see Fig. 1). According to Binford et al. (1997), a prolonged period of drought (1100–1400 A.D.) caused the decline of agricultural production and subsequently field abandonment, and this led to the society's collapse. The Tiwanaku's demise, according to Binford et al. (1997) and Delclaux et al. (2007), was due to the lack of water draining into their raised-fields. In fact, the epicenter of the Tiwanaku civilization was around Minor Lake, a lake that was altered by climatological changes (see Fig. 1). Another plausible scenario suggests that the decay of water levels may be due to water conflicts between the Wari and the Tiwanaku (Williams, 2002). Generally speaking, climate variations played a major role in other archaeological cases. Jimenez-Espejo et al. (2007) and Finlayson (2004) conjecture that Neanderthal extinction was due mainly by the last glaciation (50 000–12 000 years ago). In this case the alternative conjecture for extinction was competition with Modern Humans (Banks et al., 2008; Flores, 2011).

## 2. Model for the Tiwanaku collapse

In our mathematical model we consider two basic dynamic variables; the number of inhabitants of the extinct Tiwanaku society, and the main resource of water. We simplify the problem by assuming that the primary resources are directly related to the quantity of water present during the epoch of Tiwanaku society in the Titikaka lake, particularly in the Minor Lake (Fig. 1). This is a reasonable assumption since, as we said before, all the economic and social activities of this lost civilization were deeply related to their water resource management. In our model the key variable

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doi:10.1016/j.jtbi.2011.09.018

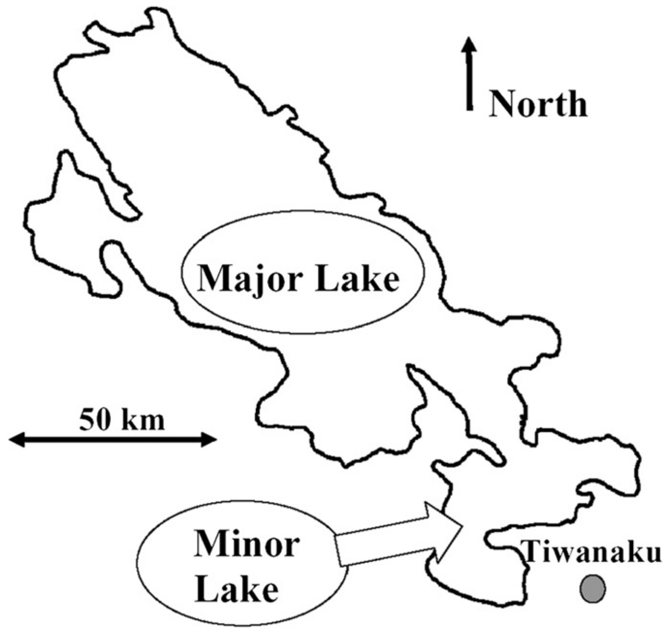


**Fig. 1.** A schematic drawing showing the actual Titikaka lake in South America, of two basins, Major Lake and Minor Lake. Tiwanaku's epicenter is marked with a dot (not scaled to size). One can also see the thin link connecting both lakes (*Estrecho de Tiquina*).

is the extension of Minor Lake, denoted by $S$, which is directly related to the quantity of water. We will assume that the collapse of the Tiwanaku civilization was due to the contraction of the surface related to the aforementioned drought and to their agricultural needs. The other dynamic variable of the system, denoted by $N$, is the number of inhabitants living in the city around Minor Lake. To determine the number of inhabitants $N$, we will consider a logistic equation (Murray, 2002) with a growth rate of $r=0.01\ \text{yr}^{-1}$). This value was obtained by Fort and Mendez (1999) and used in other models of collapsed cultures (Eastern Island and Maya of Copan, Bologna and Flores, 2008). The carrying capacity of the system will be assumed depending on accessibility to the water. The assumption is that the carrying capacity is proportional to a linear function of the extension of the lake, or more precisely, the exploited surface $S$ of the lake by the inhabitants. This hypothesis is in agreement with the model of Eastern Island in Bologna and Flores (2008) although we must point out that other valid possibilities could be considered. In reference to this paper, a valid alternative is represented by considering as a resource the volume of water instead of the surface $S$. Explicitly, we will consider the first order differential evolution equation for the number $N$ of inhabitants

$$\frac{d}{dt}N = rN\left(1-\frac{N}{R_o S}\right), \tag{1}$$

where $R_o$ is an inverse area-dimensional parameter that will be determined by using archaeological data. Archeological studies show that the level of water fell abruptly causing serious problems for the population since the carrying capacity is a function of the water. This is not a unique phenomenon. For example, as reported in Axtell et al. (2002), the collapse of the Anasazi civilization in North America was triggered by a drought from approximately the same period (1200 A.D.). Curiously, the collapse of the Easter Island civilization was also triggered around this date (Bologna and Flores, 2008).

The above equation must be coupled with the dynamic evolution of the quantity of water of Minor Lake, volume $V$. The quantity of water is given by the matter-balance equation

$$\frac{d}{dt}V = f - \alpha S - d_o N, \tag{2}$$

where $f$ is the total flux of water within the lake from rainfall, rivers, and other sources. Nevertheless, since Minor Lake is connected to Major Lake via the *Estrecho de Tiquina* (Fig. 1), a detailed knowledge of it is difficult to obtain. Despite this fact, and with respect to the surface variation, the available archaeological data allows a rough estimation of this flux change. In Eq. (2), $\alpha$ is the evaporation rate parameter, evaluated approximately as $\alpha = 1.6 \times 10^{-3}$ km/yr (Delclaux et al., 2007), $S$ is the total surface of Minor Lake and $d_o$ is the rate parameter representing the water that was drained due to the needs of the population.

The system of coupled differential equations described by Eqs. (1) and (2) has two equilibrium points. The unstable (saddle point) given by

$$N_{un} = 0, \quad S = \frac{f}{\alpha}, \tag{3}$$

which corresponds to the absence of human activity around the lake and the stable point

$$N_{st} = R_o S, \quad S = \frac{f}{\alpha + d_o R_o}. \tag{4}$$

From Eq. (4) we deduce a relation between the number of inhabitants and the area of Minor Lake. Indeed, it is closely related to the archeological data and the estimation found in this paper. If, due to climatological variations, we assume a change in the total water flux $f \to f'$, then the lake surface must also change $S \to S'$. Under these circumstances, the number of individuals will be adjusted to the constriction of the lake $N_{st} \to N'_{st}$. Explicitly, from the first of Eq. (4) we have

$$\frac{N'_{st}}{N_{st}} = \frac{S'}{S}. \tag{5}$$

This is quite an important relation since archaeological knowledge about the surface variations becomes directly related to the population change. In the same way, the total flux variation ($f \to f'$) is related to the surface change (see Eq. (4)) by $f'S = fS'$.

## 3. Evaluation from archaeological data: prediction of inhabitants collapse

Titikaka lake is composed of two basins (Fig. 1): Major Lake, with a modern approximative maximum depth of 283 m and a surface of the order of 7000 km$^2$; and Minor Lake, or *Wiñaymarka* lake, with a modern maximum depth of 41 m and a surface of 1400 km$^2$. These basins are connected by a thin link (*Estrecho de Tiquina*). In our model, and from an agricultural point of view, we assume that the inhabitants of Tiwanaku were essentially related with Minor Lake due to the closeness of their population core area with its basin, while the level of the entire lake was governed by a physical water balance. The lake level and the climatological situation before the drought were very similar to modern days (Abbott et al., 1997; Binford et al., 1997). The estimated minimum values at the epoch of collapse are 5800 km$^2$ for Major Lake and 60 km$^2$ for Minor Lake (Binford et al., 1997).

The number of persons before the collapse is widely debated. Nevertheless, according to Tainter (2007), the maximal population of Tiwanaku's city core was around 45 000 inhabitants. The evaluation of the parameter $R_o$ comes directly from Eq. (4), $R_o = N/S = 45\,000/1400 \sim 32\ \text{km}^{-2}$. From Eq. (5), and using the aforementioned data for the surface variations, we can evaluate the number of individuals after the collapse, as $N' = (60/1400) \times 45\,000 \sim 2000$ inhabitants. This is the main result of this work.