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A novel arterial constitutive model in a commercial finite element package: Application to balloon angioplasty

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ABSTRACT

Recently, a novel linearized constitutive model with a new strain measure that absorbs the material nonlinearity was validated for arteries. In this study, the linearized arterial stress–strain relationship is implemented into a finite element method package, ANSYS, via the user subroutine USERMAT. The reference configuration is chosen to be the closed cylindrical tube (no-load state) rather than the open sector (zero-stress state). The residual strain is taken into account by analytic calculation and the incompressibility condition is enforced with Lagrange penalty method. Axisymmetric finite element analyses are conducted to demonstrate potential applications of this approach in a complex boundary value problem where angioplasty balloon interacts with the vessel wall. The model predictions of transmural circumferential and compressive radial stress distributions were also validated against an exponential-type Fung model, and the mean error was found to be within 6%.

1. Introduction

The Finite Element Method (FEM) has been extensively used to analyze the mechanics of blood vessels (Perktold and Rappitsch 1995; Delfino et al., 1997; Bathe and Kamm 1999; Berry et al., 2002; Gasser et al., 2002; Holzapfel et al., 2002; Chua et al., 2004; Raghavan et al., 2004; Zhang et al., 2004; Liang et al., 2005; Liu et al., 2008). There are still some practical issues, however, that need further investigations. For instance, shell theory (Perktold and Rappitsch 1995; Berry et al., 2002) is not sufficient for thick-walled vessels, and isotropic models (Delfino et al., 1997; Bathe and Kamm 1999; Chua et al., 2004; Raghavan et al., 2004; Liang et al., 2005) are first-order approximation that do not reflect the anisotropy of vascular tissue. Gasser et al. (2002) and Holzapfel et al. (2002) implemented a microstructure-motivated constitutive model, but their geometric model of vessel selects the zero-stress state as the reference configuration which is not always convenient. Zhang et al. (2004) incorporated the Fung model into ANSYS, but their analysis was limited to small shear deformation. Liu et al. (2008) analyzed the effects of myocardial constraint, but their code did not extend to complex loadings; e.g., contact between a device and vessel wall.

Among the biomedical FEM applications, several groups have analyzed balloon angioplasty to understand the interaction between the balloon and the stent (Oh et al., 1994; Gourisankaran and Sharma 2000; Holzapfel et al., 2002; Gasser and Holzapfel 2007). Oh et al. (1994) simulated the response of atherosclerotic arteries to balloon angioplasty, where the arteries were described by an isotropic material model. Gourisankaran and Sharma (2000) also used an isotropic model and investigated the stresses in the arterial wall and atherosclerotic plaque of diseased arteries induced by balloon dilatation. Holzapfel et al. (2002) simulated balloon angioplasty using a layer-specific three-dimensional anisotropic model based on *in vitro* magnetic resonance imaging of a human stenotic postmortem artery. The FEM analysis of Gasser and Holzapfel (2007) considered the balloon-induced overstretch of remnant non-diseased tissues in antherosclerotic arteries. The model took into account the multi-layer structure of the artery including media, adventitia, and plaque; residual stresses; and plastic response of the tissue beyond the elastic limit as well.

Here, we implement a linearized anisotropic arterial constitutive relation in a commercial FEM package ANSYS v11.0 (ANSYS, Inc.). The linearized relation has the advantage of having less material parameters than the general anisotropic constitutive relation of Fung's type, which makes it easier to determine the parameters from experimental data. The advantage of commercial FEM package is that general boundary value problems having complex contact and fluid–solid interaction are readily solvable.

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Illustratory examples of balloon (compliant and noncompliant) angioplasty involving compliant and noncompliant balloons are solved and compared based on the proposed platform. We also validate the transmural stress predictions with a Fung type model. The technical feasibility of this approach and the potential applications are envisioned.

2. Methods

2.1. Deformation and constitutive model

The mechanical property of blood vessel wall is modeled as being cylindrically orthotropic, elastic and nearly incompressible. The zero-stress state (ZSS, described by cylindrical coordinates R, Z, Θ) is a cut-open sector (Fig. 1(a)), and the deformed configuration (described by cylindrical coordinates r, z, θ) is a closed circular tube (Fig. 1(b)). For convenience, the reference configuration is selected as the no-load state where the vessel is intact and subjected to zero blood pressure and no axial stretch. Therefore, there is a residual deformation gradient from ZSS to the reference configuration, as will be given later. At ZSS, the inner and outer circumferences, L_i and L_o, and the cross-sectional wall area A_0 are measured. The inner and outer radii of the open sector R_i and R_o, and the opening angle Φ are

$$R_i = \frac{\chi L_i}{2\pi}, \quad R_o = \frac{\chi L_o}{2\pi}, \quad \Phi = \pi - \frac{L_o^2 - L_i^2}{4A_0}, \quad \left(\chi = \frac{\pi}{\pi - \Phi}\right) \tag{1}$$

Following the coordinate ordering in ANSYS, we relate the deformed configuration (r, z, θ) to the cylindrical reference configuration (ξ, ζ, ψ) by

$$r = r(\xi,\zeta), \quad z = z(\xi,\zeta), \quad \theta = \psi \tag{2}$$

Thus the total deformation gradient $\mathbf{F}$ (referring to ZSS) is calculated as the tonsorial product of the deformation gradient from the reference configuration to the current deformed configuration, $\mathbf{F}_1$, and that from ZSS to the reference configuration, $\mathbf{F}_0$, as

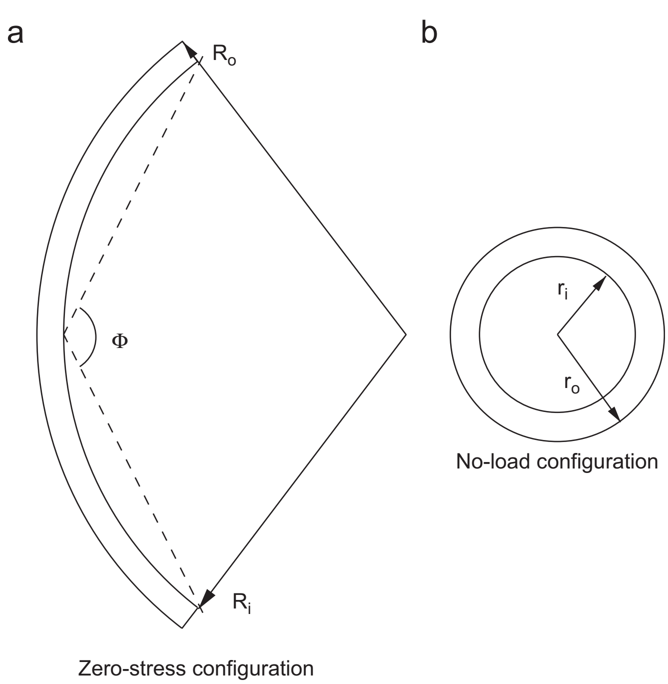

Fig. 1. The zero-stress state (a) and deformed state (b) of the arterial vessel. The reference configuration is a deformed state without pressure and axial stretch, i.e., the no-load state.

$\mathbf{F} = \mathbf{F}_1 \cdot \mathbf{F}_0$. The matrix form is

$$\mathbf{F} = \mathbf{F}_1\mathbf{F}_0 = \begin{bmatrix} \partial r/\partial\xi & \partial r/\partial\zeta & 0 \\ \partial z/\partial\xi & \partial z/\partial\zeta & 0 \\ 0 & 0 & r/\xi \end{bmatrix} \begin{bmatrix} \Lambda_r & 0 & 0 \\ 0 & \Lambda_z & 0 \\ 0 & 0 & \Lambda_\theta \end{bmatrix} \tag{3}$$

where Λ_r, Λ_z and Λ_θ are the principal stretch ratios in the reference radial, axial and circumferential directions, respectively, which can be analytically calculated from the incompressibility condition (Zhang et al., 2004; Rachev et al., 1996), as

$$\Lambda_r = \frac{\partial r}{\partial R} = \frac{R}{\chi \Lambda_z r}, \quad \Lambda_\theta = \frac{\chi r}{R} \tag{4}$$

in which Λ_z equals to the prescribed axial stretch, and the relation between the reference and ZSS radial coordinates, r and R, respectively, is

$$R = \sqrt{R_i^2 + (r^2 - r_i^2)\chi\Lambda_z} \tag{5}$$

where r_i is the inner radius of the no-load reference configuration. A substitution of Eq. (5) into (4) provides $\mathbf{F}_0$ expressed analytically in terms of R_i, χ, Λ_z, r_i, and the radial coordinate, r, in the no-load configuration. In ANSYS USERMAT, $\mathbf{F}_0$ is calculated and stored at every integration point.

To define strain, $\mathbf{F}$ is usually decomposed (Ogden 1984) as

$$\mathbf{F} = \mathbf{RU} \tag{6}$$

and the right Cauchy-Green deformation tensor is calculated as:

$$\mathbf{C} = \mathbf{F}^T\mathbf{F} = \mathbf{U}^2 \tag{7}$$

In the constitutive model of vessel wall used in this analysis, we employ a logarithmic-exponential (log-exp) strain tensor, which is defined to absorb the material nonlinearity (Zhang et al., 2007b)

$$\mathbf{D} = \exp[n(I_1 - 3)]\ln\mathbf{U} \tag{8}$$

where n is a nonlinearity parameter, and I_1 the first invariant of $\mathbf{C}$

$$I_1 = \mathrm{tr}(\mathbf{C}) = \mathrm{tr}(\mathbf{F}^T\mathbf{F}) = \mathrm{tr}(\mathbf{U}^2) \tag{9}$$

Correspondingly, a co-rotational Cauchy stress $\mathbf{T}$ was introduced (Zhang et al., 2007b), as

$$\mathbf{T} = \mathbf{T}^e + H\mathbf{I} \tag{10}$$

where H is the negative hydrostatic pressure, and $\mathbf{T}^e$ is the deviatoric stress due to deformation, and is related to the deviatoric Cauchy stress $\boldsymbol{\sigma}^e$ by $\boldsymbol{\sigma}^e = \mathbf{R}^T\mathbf{T}^e\mathbf{R}$.

In our analyses (Zhang et al., 2007a, 2007b; Liu et al., in press) on the experimental data of coronary arteries in triaxial deformation (axial stretch, inflation and torsion) (Lu et al., 2003), it was found that the log-exp strain $\mathbf{D}$ and the deviatoric co-rotational Cauchy stress $\mathbf{T}^e$ can be closely fit by a generalized Hooke's law $\mathbf{D} = \mathbf{M}:\mathbf{T}^e$, where $\mathbf{M}$ is a constant forth-order compliance tensor as for the classical Hooke's law of linear elastic materials. For blood vessel, $\mathbf{M}$ satisfies incompressibility condition, $\mathbf{M}:\mathbf{I} = \mathbf{0}$ such that hydrostatic pressure does not induce any deformation. In axisymmetric deformation, as in the present simulation of balloon angioplasty, the generalized Hooke's law can be expressed in a matrix form, as

$$\begin{pmatrix} D_{\theta\theta} \\ D_{zz} \\ D_{rr} \\ 2D_{zr} \end{pmatrix} = [M] \begin{pmatrix} T^e_{\theta\theta} \\ T^e_{zz} \\ T^e_{rr} \\ T^e_{zr} \end{pmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{12}/E_1 & -\nu_{13}/E_1 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & -\nu_{23}/E_2 & 0 \\ -\nu_{13}/E_1 & -\nu_{23}/E_2 & 1/E_3 & 0 \\ 0 & 0 & 0 & 1/G_{23} \end{bmatrix} \begin{pmatrix} T^e_{\theta\theta} \\ T^e_{zz} \\ T^e_{rr} \\ T^e_{zr} \end{pmatrix} \tag{11}$$

where$[M]$ is the matrix representation of the compliance tensor $\mathbf{M}$. The parameters E_1, E_2 and E_3 can be interpreted as Young's moduli in reference to the log-exp strains, G_{23} is the shear modulus, and ν_{12}, ν_{13} and ν_{23} are Poison's ratios. Unlike the general orthotropic materials where the six parameters E's and ν's in Eq. (11) are independent, the incompressibility condition $\mathbf{M}:\mathbf{I} = \mathbf{0}$ imposes