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A mechanism of dynamical interactions for two-person social dilemmas

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ABSTRACT

We propose a new mechanism of interactions between game-theoretical agents in which the weights of the connections between interacting individuals are dynamical, payoff-dependent variables. Their evolution depends on the difference between the payoff of the agents from a given type of encounter and their average payoff. The mechanism is studied in the frame of two models: agents distributed on a random graph, and a mean field model. Symmetric and asymmetric connections between the agents are introduced. Long time behavior of both systems is discussed for the Prisoner's Dilemma and the Snow Drift games.

1. Introduction

In mathematical modeling of human societies the evolution of behaviors (actions) of the interacting individuals and of the structure of the mutual interactions should be taken into account. In real world the acceptance and refusal of entering in interactions with preferential partners is a common phenomenon. Real world networks change in time. People may tend to regulate the interpersonal interactions, connections with the others, on the basis of comparison between the results of the interactions and some averaged, local or global patterns. The players can change their actions and optimize the strength of relations with the others. In particular the players can change the structure of the links with the other players, breaking links and creating the new ones.

The models of populations of agents with interactions described by social dilemma games on spatial static regular lattices were introduced by Nowak and May (1992), and then studied by many authors, and extended to general spatial networks and structured populations. We refer to Szabo and Fath (2007), and Gross and Blasius (2008) for recent reviews on evolutionary games on graphs and on adaptive coevolutionary networks.

The coevolution of network topology and strategy dynamics has been considered by many authors. Various concepts of the coevolving network structure and strategy distribution have been introduced, e.g. via assortative selection of interaction partners (cf. e.g. Ashlock et al., 1996; Ebel and Bornholdt, 2002; Eguiluz et al., 2005; Zimmermann and Eguiluz, 2005; Poncela et al., 2007, and references cited therein), volunteering participation (cf. e.g. Hauert and Szabo, 2003, and references cited therein), via random or intentional rewiring procedures (e.g. addition and/or removal of nodes, cf. e.g. Zimmermann et al., 2004), via introduction of different behaviors towards the adverse ties (Van Segbroeck et al., 2008, 2009) and by introducing active linking and agent-based linking dynamics (Pachecho et al., 2006a, b; Traulsen et al., 2008). In particular Pachecho et al. (2006a, b) considered a population model in which the agents seek new connections at different rates, and allow the established connections to last for different amount of time. In the limit in which the dynamics of the network is much faster than the evolution of strategies the authors in particular show that the Prisoner's Dilemma (PD) game can be transformed to a coordination game—the transformation changes the rules of the game and explains the emergence of cooperation in the considered model. Pachecho et al. (2008) studied the systems with repeated interactions which last as long as the link between the players is present, and obtained analytical conditions for evolutionary stability under direct reciprocity. In comparison with the similar model on static graph (Ohtsuki and Nowak, 2007), the cooperation is facilitated by the active linking dynamics (in both cases the cooperation is promoted if the links last long enough, and the incentive to create new links is not too high).

We propose an approach, based on another idea of changing the connectivity structure in the system. We assume that the players dynamically change the connection weights, using update rules which reflect the tendency to increase more profitable connections, and to weaken the disadvantageous ones in a continuous way. The weights can be symmetric or asymmetric. In the first case the value of the connection is the same for both players, whereas in the second this value can be different for each of them.

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We propose two types of models: finite population on a network and continuous population in the mean field approximation. In the model of agents on network the agents are located on a random graph. Each agent is connected with some other agents (neighbors). Each connection has a weight, which changes according to preferences of the agent. The preferences of a player are measured by the difference between its payoff from the considered connection and an averaged payoff. Evolution of the model occurs by a birth–death (BD) mechanism. In the mean field model the weights of the connections are time-dependent functions which evolve according to the rules of evolutionary game theory. Solutions of the resulting systems of differential equations are discussed. The asymptotic equilibrium states are investigated. For both models we study temporal evolution of the strategies and the weights distribution for two types of two-person games describing the standard social dilemmas: the Prisoner's Dilemma game and the Snow Drift (SD) game.

2. Model on graph

The population consists of N agents, identified with the nodes of a random connected graph with a degree of k. The edges are described by dynamically changing connection weights: we denote $\omega_{ij}(t)$—the connection weight between the nodes i and j at time t. The connection weights can be symmetric: $\omega_{ij}(t) \equiv \omega_{ji}(t)$, or asymmetric, when $\omega_{ij}(t)$ and $\omega_{ji}(t)$ are in general different. The agents interact pairwise, playing a two-person symmetric game with the payoff matrix

	C	D
C	a	b
D	c	d

(abbreviated in the description of figures by $[a,b,c,d]$) with their neighbors, using the strategy C or D, and receive payoffs which are products of the payoffs from the above payoff matrix, and the relevant connection weights. We shall refer to such products as to the *effective payoffs*. In our paper C stands for cooperation and D for defection in the considered below social dilemma games, although in general the proposed scheme is valid for any two-person game, and can be generalized for other types of the games.

Initially the agent's strategies are allocated randomly. The initial connection weights are allocated in such a way that the assumed order of the graph is obtained, and typically are the same for all connected players.

The evolution of the network takes place in discrete time steps. At each time step first the weights of all the connections are updated, then the strategy of one of the agents and its weights are updated.

The updating rules will reflect the fact that each player tends to increase the intensity of interactions (the connection weights in our model) with those opponents with whom the results of the interactions, i.e. the effective payoffs are higher than the average payoff from all the interactions of the player. The increase is proportional to the difference between both types of payoffs. The rules of the weights updating in the symmetric and asymmetric models are, respectively,

$$\omega_{ij}(t+1) = \omega_{ij}(t) + \omega_{ij}(t)(1-\omega_{ij}(t))(\Delta_{ij}+\Delta_{ji})/2M \quad (1)$$

$$\omega_{ij}(t+1) = \omega_{ij}(t) + \omega_{ij}(t)(1-\omega_{ij}(t))\Delta_{ij}/M \quad (2)$$

where Δ_{ij} is the difference between the effective payoff of the agent in node i from the interaction with that in j (given by the product of ω_{ij} and the relevant entry of the payoff matrix introduced above), and the mean payoff of the agent in i over the neighborhood of i. Formally Δ_{ij} is defined as follows. Let d_{ij} denote the payoff of the i player from the interaction with the j player, calculated from the relevant entry of the initial payoff matrix (1). Let N_i denote the number of the i-th neighbors, i.e. the nodes connected to i. Then $\Delta_{ij}(t) = \omega_{ij}(t)d_{ij} - (1/N_i)\sum_{l=1}^{l=N_i}\omega_{il}(t)d_{il}$. M denotes the maximum of the payoff matrix.

The product $\omega_{ij}(1-\omega_{ij})$ in (1) and (2) reduces the speed of the evolution of the relevant weight when it approaches its extremal values (here normalized to zero and unity). In other words, the ties which are close to their extremal values are harder to change.

Strategy updating for both the symmetric and antisymmetric weight models is based on the BD method (Ohtsuki and Nowak, 2006). We consider two types of updates, which we call no inheritance and inheritance updates. In the former we draw a player (parent), with probability proportional to its total payoff. Then we draw randomly one of its neighbors (descendant), and allocate to it the parent's strategy. The weights of the descendant with its neighbors are set to their initial values. In the latter the strategy is inherited as above; the connection weights between the neighbors and the descendant are calculated from the neighbor's and the parent mean values. Each player can have two, in general different, mean values of the connection weights: one calculated from the interactions with partners who play C and one with those who play D. In the inheritance symmetric model the weights neighbors–descendant are equal to the relevant mean values of the parent. In the inheritance asymmetric model the weights descendant–neighbors are equal to the mean values of the parent's weights, the weights neighbors–descendant are for each neighbor equal to the relevant mean values of the neighbor's weights.

The proposed model belongs to the class of the coevolutionary models with two characteristic time scales: the first one characterizes the frequency of strategy updates, the second one describes the frequency of changes of the weights. In many biological applications it has been assumed that the time scale of the interactions between the individuals is much shorter than that of the time scale of the selection processes. The dependence of the results, in particular of the maintenance of coordination in the long run, on the scaling of these two processes was studied by various authors. In particular Santos et al. (2006) show that for a given average connectivity of the population, there is a critical value of the ratio of the time scale associated with the evolution of strategies to the network connectivity structure, above which the cooperation is maintained in the system. The problem of time scales for different processes in the coevolutionary models was also considered e.g. by Pachecho et al. (2006a, b) and Roca et al. (2006). Our analysis is restricted to the situations in which the characteristic time scale of the strategy updating is much bigger than that of the connection updating.

3. Mean field model

We assume the evolutionary scenario in which each agent interacts with the other agents through a random pairwise matching, playing at each instant of time a two-person symmetric game with the payoff matrix $[\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}]$.

Let $\mu = \mu(t)$ denote the frequency of agents playing C in the whole population. We introduce $\omega_{FS} \equiv \omega_{FS}(t)$—the weight of the connection between the agent playing strategy F and that playing S and U_{FS} the effective payoff of the F-agent from the interaction with the S-agent, $F, S \in \{C, D\}$:

$$U_{CC} = a\omega_{CC}, \quad U_{CD} = b\omega_{CD}, \quad U_{DC} = c\omega_{DC}, \quad U_{DD} = d\omega_{DD} \quad (3)$$