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Aggregation is the key to succeed in random walks

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ABSTRACT

In a random walk (RW) in $\mathbb{Z}$ an individual starts at 0 and moves at discrete unitary steps to the right or left with respective probabilities p and $1-p$. Assuming $p > 1/2$ and finite a, $a > 1$, the probability that state a will be reached before $-a$ is $Q(a, p)$ where $Q(a, p) > p$. Here we introduce the cooperative random walk (CRW) involving two individuals that move independently according to a RW each but dedicate a fraction of time θ to approach the other one unit. This simple strategy seems to be effective in increasing the expected number of individuals arriving to a first. We conjecture that this is a possible underlying mechanism for efficient animal migration under noisy conditions.

1. Introduction

This problem is inspired by animal movement. The question we address is how a group of individuals can manage to increase their chances to arrive to the correct destination by traveling in groups. Mathematical models that propose underlying mechanisms for this phenomena, may shed some light on the still open question on the *addition of intelligence* that would explain why animals benefit from traveling in groups, even with limited communication skills. Here we define limited communication skills as an individual's ability to detect only the position of the other, either visually or by contact; that is, individuals can somehow share the decision taken, but not the rationale that led to that decision.

In our quest for a simple model, we start by assuming that (a) our population is composed by two individuals that are equally prepared to process information from the environment, (b) that this information relates to the correct travel direction, and (c) the processing of information is carried out independently by each individual. Underlying this process, there is the sense of grouping that compels the individuals to remain together, that is, they act independently most of the time, but an underlying aggregation force compels them no to go far away from each other. These simple conventions will be the basis for our model.

To analyze this phenomena, we chose as a model the *Random Walk*, a very versatile class of models. The term was coined by Pearson [11] and has been used in fields such as biology, chemistry, computer science, ecology, economics, physics, psychology [13,14] and in particular, to model the motion of animals [2,6,7,10]. We chose the random walk with boundaries, where at each step, an individual moves to the right with probability p and to the left with probability $1-p$. In our model, $p > 1/2$ and the individuals succeeds if the individual reaches some positive state a before $-a$. With this model, we can pose the problem as follows: under what mechanism can two individuals collaborate to increase their individual chances of arriving at the correct destination a when their communication skills are reduced to a minimum?

2. The random walk with absorbing barriers

In a random walk with absorbing barriers in $\mathbb{Z}$ an individual moves along the lattice

$$\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3 \ldots \pm a\}$$

moving to the right at each time step with probability p or to the left with probability $1-p$. The process stops when the individual reaches $\pm\, a$. It is known as *gambler's ruin problem*. If a is the correct destination we can assume the individual is more prone to move in that direction so $p > 1/2$. An individual starting at 0 will be absorbed eventually at $-a$ (ruin) or a (win). The probability of winning is:

$$Q(a, p) = \left[1 + \left(\frac{1}{p} - 1\right)^a\right]^{-1},$$

which is increasing in a [see 5, pp. 345–346].

3. The cooperative random walk

In an effort to model cooperation, we introduce the *Cooperative Random Walk* (CRW). The position at time t are $X(t)$ and $Y(t)$ for each individual. In our model, at time t one individual is selected at random to move. This individual, with probability $1-\theta$ moves according to a random walk and with probability θ moves one unit

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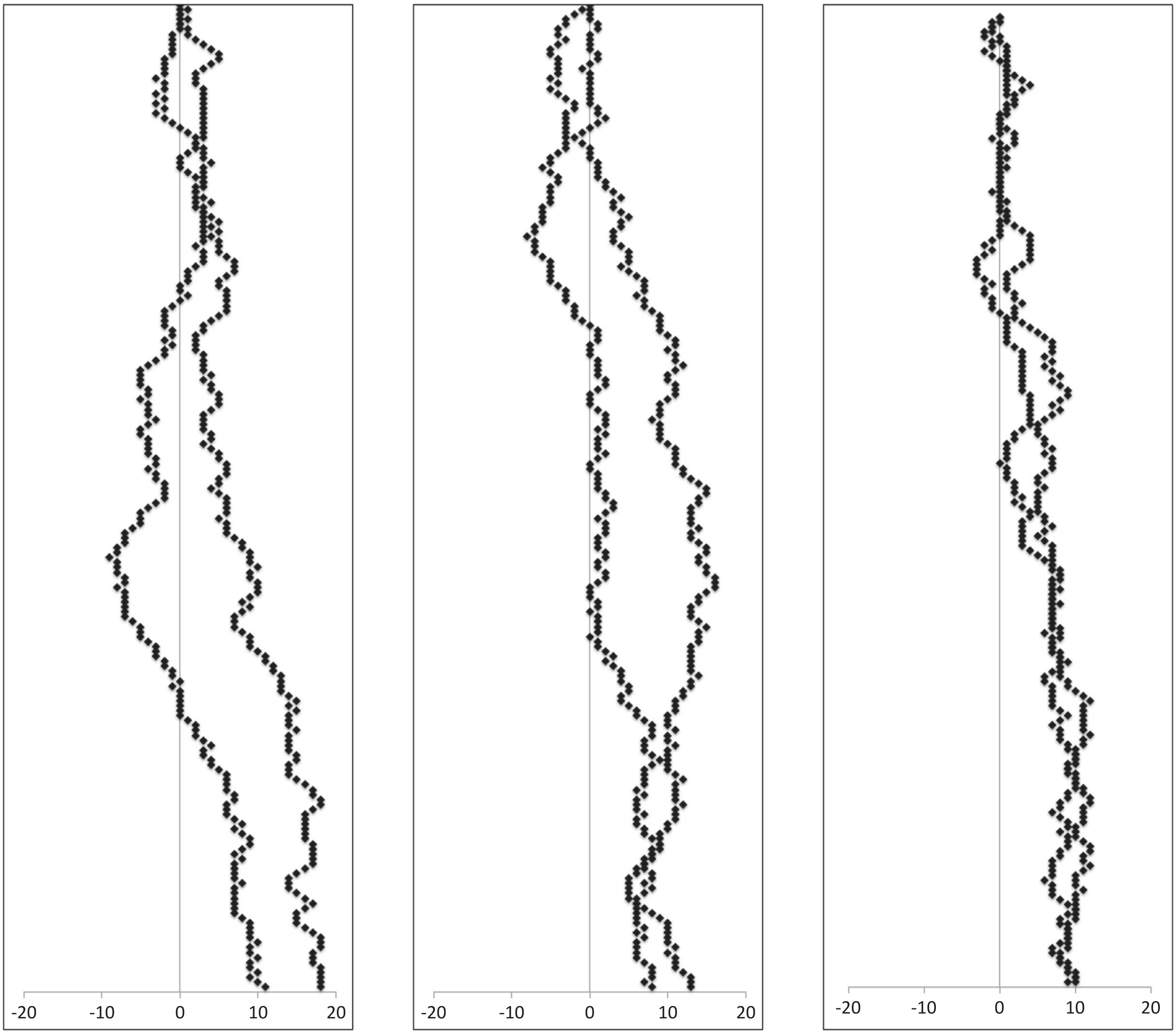

Fig. 1. Progression of the position of both individuals after 200 movements, for $p = 0.53$ and θ values of 0, 0.1 and 0.2 Step number runs across the vertical axis (top to bottom). Both individuals start at 0.

towards the position of the other. Thus, θ plays the role of the *aggregation* effect. In our model we assume that if $X(t) = Y(t)$, then both individuals will stay in the same position with probability θ or with probability $1 - \theta$ one of them will move according to a random walk.

While in the random walk, the process stops when an individual reaches one of the boundaries, in the CRW, we do not stop the movement of an individual when it reaches one of the boundaries since then its position is fixed and would attract the other to its position permanently. To avoid this effect, we modified the process so that it stops the first time both individuals are outside the interval $(-a, a)$, that is, the process stops at the minimum t such that

$$|X(t)| \geq a, \quad |Y(t)| \geq a$$

The CRW model lies within the realm of game theory [3,12]. *Cooperative* games are defined as games in which groups of players are formed and organize in some way to take a decision that will lead the group to win or lose [4,9]. *Sequential* games are games in which the outcome of the game is defined only after a series of alternating decisions between players [1,8]. Although both fields are vast and a lot of literature has been published on these topics separately, especially on non-cooperative sequential games, literature is scarce regarding models of sequential decisions taken by cooperating individuals. It is relevant to differentiate between the *Cooperative Random Walk* with the *correlated Wiener process* [see 15]. In the correlated Wiener process, two individuals at positions $X(t)$ and $Y(t)$ at time t change their position at time $t+1$ to $X(t) + x$ and $Y(t) + y$, respectively, where x and y are correlated random variables. The parameter ρ measures the correlation between x and y. When $\rho = 1$ then $x = y$ and the distance between both individuals is constant, equal to their initial distance, that is, $X(0)$ and $Y(0)$. On the other hand, in the CRW when $t \to \infty$, an aggregation effect of 1 would imply that regardless of the initial amount of separation, both individuals tend to be together. This is an important difference: although each model implicitly assumes some amount of communication, in the correlated Wiener processes, the aggregation effect is more a *mimicry* effect that reflects the similarity of the direction and amount of motion of both individuals, thus, the correlation coefficient ρ may be used to model the amount of