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Mathematical model for blood flow through a bifurcated artery using couple stress fluid

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ABSTRACT

In this article, the blood flow through a bifurcated artery with mild stenosis is investigated taking blood as couple stress fluid. The artery configuring bifurcation is assumed to be symmetric about the axis of the artery and straight cylinders of finite length. The governing equations are non-dimensionalized and coordinate transformation is used to convert the irregular boundary to a regular boundary. The resulting system of equations is solved numerically using the finite difference method. The variation of shear stress, flow rate and impedance near the apex with pertinent parameters are studied graphically. It has been noticed that shear stress, flow rate and impedance have been changing suddenly with all the parameters on both sides of the apex. This occurs because of the backflow of the streaming blood at the onset of the lateral junction and secondary flow near the apex in the daughter artery.

1. Introduction

The cardiovascular system is an internal flow loop with multiple branches in which a complex liquid circulates. Atherosclerosis, localized narrowing of the blood vessels due to abnormal growth of tissues, is one of the diseases of the cardiovascular system of man. This can cause serious circulatory disorders by reducing or occluding the blood supply. The early atherosclerotic changes of vessel walls and the deposition of platelet thrombi occur preferentially at the entrances of branching arteries [1]. Thus, over the last two decades, extensive research has been carried out to correlate the connection between blood flow, localized genesis and development of atherosclerosis, including experimental, analytical and numerical studies. Studies for both normal and stenotic vessels have been carried out for idealized arteries, idealized arterial bifurcations and branching.

In most of the studies reported in the literature on the blood flow dynamics, blood is assumed to be an incompressible Newtonian fluid. It is well known that most physiological fluids, including blood behave as a non-Newtonian fluid. It has been suggested that blood flow behavior in narrow vessels at low shear rate can be represented by a non-Newtonian fluid. The non-Newtonian behavior of blood is mainly due to the suspension of red blood cells in the plasma. When neutrally buoyant blood cells are contained in a fluid and there exists a velocity gradient due to shearing stress, blood cells have rotatory motion have spin angular momentum in addition to linear angular momentum. Hence, the symmetry of the stress tensor is lost in the fluid motion that is subjected to spin angular momentum. The radius of gyration of the blood cells in such fluids is different from that of the fluid particles. Their difference produces couple stress in the fluid. It is known that such a fluid has spin angular momentum in addition to the couple stress effect. The couple stress fluid model introduced by Stokes [2] has distinct features, such as the presence of couple stresses, body couples and non-symmetric stress tensor. The main feature of couple stresses is to introduce a size dependent effect. Classical continuum mechanics neglect the size effect of material particles within the continua. These fluids are capable of describing various types of lubricants, blood, suspension fluids, etc.

Mekheimer and Abd Elmaboud [3] discussed the peristaltic flow of a couple stress fluid in an annulus with the application of an endoscope and observed that the trapped bolus decrease in size as one move from couple stress fluid to a Newtonian fluid. Sobh [4] analytically studied the interaction of couple stresses and slip flow on peristaltic transport in a uniform and non-uniform channels and concluded that the peristaltic pumping for the couple stress fluid is more than that of Newtonian fluid. Sahu et al. [5] presented a mathematical analysis to study the effect of a mild stenosis on blood flow characteristics with the representation of blood by couple stress fluid. Nadeem and Akram [6] considered the peristaltic flow of the couple stress fluid in an asymmetric channel under the influence of the induced magnetic field and noticed

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that the pressure rise increases in the peristaltic pumping region to increase in couple stress parameter. Pandey and Choube [7] examined the effect of magnetic field on the peristaltic transport of couple stress fluids through a porous medium and observed that the mean velocity at the channel walls decreases with an increase in the value of couple-stress, permeability parameters and it increases with an increase in the magnetic parameter. Sankad and Radhakrisnamacharya [8] investigated that the time average velocity decreases with viscous damping force and the size of the trapped bolus and decreases with the Hartmann number for the peristaltic transport of couple stress fluid in a channel with different wall properties. Maiti and Misra [9] investigated that the increase in both pumping and pressure by increasing the amplitude ratio, couple stress parameter and also by decreasing the permeability for the peristaltic transport of a couple stress fluid in a porous channel. Srinivasacharya and Srikanth [10] studied the steady streaming effect on the pulsatile nature of couple stress fluid and analyzed that the pressure drop, the shear stress increases with an increase in the size of the catheter and decrease in the couple stress fluid parameter. Tripathi [11] analyzed the peristaltic pumping properties, frictional force, mechanical energy and trapping occurrence for peristaltic hemodynamic flow patterns and concluded that mechanical energy is a decreasing function of couple-stress parameter. Akbar and Nadeem [12] mentioned that the pressure rise increases with an increase in amplitude ratio, couple stress fluid parameter and decrease in thermophoresis parameter for the peristaltic flow of an incompressible couple stress fluid in a two-dimensional uniform tube. Hayat et al. [13] investigated the hall effects on the peristaltic flow of couple stress fluid in an inclined asymmetric channel with heat and mass transfer. They noticed that the size of the trapping bolus decreases with an increase in the value couple stress parameter, Hall parameter and decreases in the value of Hartmann number. Akbar and Nadeem [14] calculated the exact solutions for the peristaltic flow of chyme in intestine using couple stress fluid. Alsaedi et al. [15] considered the flow of couple stress fluid through uniform porous medium and revealed that the volume of the trapped bolus increases and circulates faster with an increase in the value of permeability parameter. Hina et al. [16] reported that the fluid velocity significantly increases with an increase in the couple stress fluid parameter and decreases with an increase in the magnetic field strength for the peristaltic motion of an electrically conducting couple-stress fluid in a channel with complaint walls.

In all the above mentioned papers, the significance of the bifurcation of the arteries was neglected. The study of blood flow in the bifurcated artery is of great scientific attention with respect to the diagnosis of atherosclerosis. All the physiological properties of blood at the bifurcation of the artery severely affected by different parameters. Hence, the aim of the present article is to study the couple stress fluid flow through a bifurcated artery with mild stenosis in the parent lumen. The present study employs a two dimensional model that is tractable computationally although the related physiological situation is three dimensional. The variation of volumetric flow, impedance and shearing stress are analyzed for various values of pertinent parameters involved in the problem.

2. Mathematical formulation

Consider the flow of steady, laminar, incompressible homogeneous blood flow through a bifurcated artery with mild stenosis in its parent lumen. The blood is treated as couple stress fluid of constant density. The stenosis over a length of the artery is assumed to have developed in an axi-symmetric manner and the parent aorta have a single mild stenosis in its lumen as shown in Fig 1. Let (r, θ, z) be the coordinates of the material point in the cylindrical polar coordinate system, of which z is taken as the central axis of

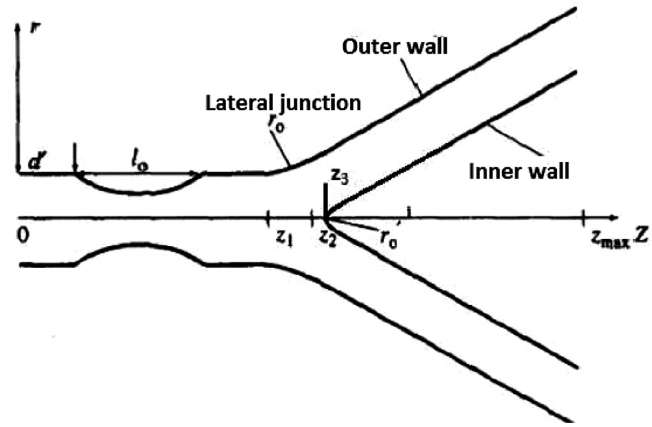

Fig. 1. Schematic diagram of stenosed bifurcated artery.

the parent artery. The arteries, forming bifurcations are symmetrical about the central axis of the parent artery and are straight circular cylinders of restricted length. Curvature is introduced at the lateral junction and the flow divider so that the flow separation zones (if any)can be removed.

The governing equations for the motion of incompressible steady couple stress fluids in the absence of body force and body moments are given by

$$\nabla \cdot \mathbf{q} = 0 \tag{1}$$

$$\rho(\mathbf{q}.\nabla)\mathbf{q} = -\nabla p + \mu \nabla^2\mathbf{q} - \eta \nabla^4\mathbf{q} \tag{2}$$

where ρ is the density of the couple stress fluid, η is the couple stress viscosity parameter, $\mathbf{q}$ is the velocity vector and μ is the viscosity.

The force stress tensor τ and the couple stress tensor $\mathbf{M}$ that are present in the theory of couple stress fluid are respectively given by

$$\boldsymbol{\tau} = (-p + \lambda\nabla \cdot \mathbf{q})\mathbf{I} + \mu\left[\nabla\mathbf{q} + (\nabla\mathbf{q})^T\right] - (1/2)\mathbf{I} \times (\nabla \cdot \mathbf{M}), \tag{3}$$

and

$$\mathbf{M} = m\mathbf{I} + 2\eta\nabla(\nabla \times \mathbf{q}) + 2\eta' \left(\nabla(\nabla \times \mathbf{q})\right)^T, \tag{4}$$

where $\mathbf{I}$ is the unit tensor, p is the fluid pressure, m is $1/3rd$ trace of $\mathbf{M}$, The quantity λ is the material constant and η and η' being the constants associated with couple stresses. The dimensions of the material constant λ are that of viscosity, whereas the dimensions of η and η' are those of momentum. These material constants satisfy the following inequalities.

$$\mu \geq 0, \quad 3\lambda + 2\mu \geq 0, \quad \eta \geq 0, \quad \eta' \leq \eta. \tag{5}$$

The mathematical representation of an outer $R_1(z)$ and the inner $R_2(z)$ walls of the bifurcated artery with mild stenosis in parent lumen are ([17,18])

$$R_1(z) = \begin{cases} a & 0 \leq z \leq d' \\ (a - \frac{4\epsilon}{l_0^2}(l_0(z-d') - (z-d')^2) & d' \leq z \leq d' + l_0 \\ a & d' + l_0 \leq z \leq z_1 \\ (a + r_0 - \sqrt{r_0^2 - (z-z_1)^2}) & z_1 \leq z \leq z_2 \\ (2r_1 sec\beta + (z-z_2)tan\beta) & z_2 \leq z \leq z_{max} \end{cases} \tag{6}$$

$$R_2(z) = \begin{cases} 0 & 0 \leq z \leq z_3 \\ (\sqrt{(r_0')^2 - (z - z_3 - r_0')^2}) & z_3 \leq z \leq z_3 + r_0'(1 - sin\beta) \\ (r_0' cos\beta + z_4) & z_3 + r_0'(1 - sin\beta) \leq z \leq z_{max} \end{cases} \tag{7}$$