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Changes in habitat of fish populations: An inverse problem

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ABSTRACT

Mathematical modelling applies to a wide variety of application areas, and is an active area of research in many disciplines. It is often the case that accurate depiction of real-world phenomena require increasingly complex models. Unfortunately, this increased complexity in a model causes great difficulty when seeking solutions. What is more, developing a model with known parameters that produces results consistent with observed behaviors may prove to be a difficult or even impossible task. These difficulties have brought about an interest in inverse problems.

In this paper we utilize a collage-based approach to solve an inverse problem for a model for the migration of three fish species through floodplain waters. A derivation of the mathematical model is presented and a generalized collage method is discussed and applied to this model to recover diffusion parameters. Theoretical and numerical particulars are discussed and results are presented.

1. Introduction

Brazil is one of the world's leading dam-building nations, with about 80% of its electrical energy coming from dams in the form of hydropower [18]. While an expansion of hydroelectricity in Brazil would lead to crucial economic growth, the social and environmental costs of constructing dams may far outweigh this economic gain. Among these environmental costs is the endangerment of wildlife and their habitats. While many studies have examined the effect of anthropogenic activity on the persistence and resilience of terrestrial organisms, very little research has focussed on submerged populations.

In this paper, we develop a nonlinear model for the dispersal of fish populations in floodplain waters, and then use a collage-based approach to solve the inverse problem presented by this model. We begin in Section 2 with a discussion of some ecological underpinnings. In Section 3 we combine the modelling ideas in [2,3,5,12] to derive a model that describes the phenomenon of floodplain migration mathematically. In Section 4 we discuss the existence and uniqueness of a weak solution to our model problem. With an understanding of the forward problem, in Section 5 we state the inverse problem of interest and discuss a collage-based method for solving this problem. One practical application of such an inverse problem is to determine which fish populations may be threatened by damming (based on movement patterns) so that they may be relocated prior to damming and thus saved. Finally in Section 6 we present results from numerical simulations for the three different movement types studied: moderate, latitudinal, and longitudinal movement. Finally we discuss the results and suggest avenues for improvement.

2. Background

A floodplain is a land area susceptible to being inundated by flood waters from any source [19]. Fig. 1 depicts the morphology of a general floodplain. Flooding of these land areas typically originates from one of three sources: overspill from the river channel, local rainfall, and/or tidal action. A number of factors contribute to fish choosing to populate these floodplains. The chemical balance of the water can make floodplains more or less desirable habitat for different species of fish. Chemical cues include the ionic composition, pH level, temperature of the water and composition of the soil. A well-balanced chemical make-up promotes the growth and sustenance of vegetation which is necessary for the livelihood of fish and other organisms. The quantity and type of vegetation that can grow in floodplains is strongly influenced by the depth of the water. Fish and other organisms may also choose to inhabit floodplains for mating and spawning, or to avoid large prey that are limited to larger bodies of water.

Fish and other organisms typically populate floodplain waters during wet seasons, perhaps travelling between the floodplain and the river to seek out food, escape predators, or spawn. The danger to the population occurs during dry seasons when water levels fall, sometimes stranding populations in shallow waters where they eventually deplete their resources and perish. In the case of

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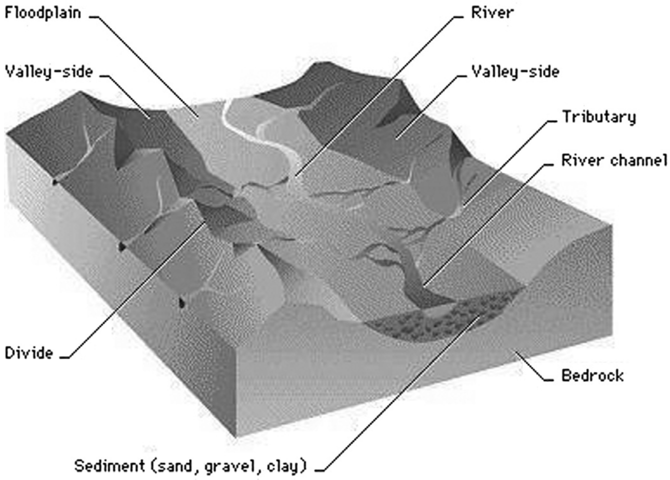

Fig. 1. Diagram of the morphology of a general floodplain environment [20].

dams, the natural flow of water is interrupted, often causing inadequate replenishment of the microorganisms and bacteria needed for vegetation growth. The absence of this natural flow of water may also prevent many fish from moving up stream to their natural breeding grounds, causing failure of breeding cycles or blockage of migration paths. By gaining a better understanding of the migratory patterns of different fish and other organisms, proper precautions may be taken to sustain these populations.

Various studies of migratory patterns (see for instance [14,17]) of fish identify three main behavioral patterns:

(i) those that exhibit moderate migration, causing them to exist in a small radius.
   - Fish in this species undertake only moderate movements within the river (mainly for dispersal to dry season habitats), but spawn in the floodplain.

(ii) those that migrate laterally seeking habitats along river borders toward the fringes of the main channel.
   - These species are largely confined to the floodplain during wet seasons and settle into residual pools and lagoons in dry seasons. Species in this group are often referred to as 'blackfish'.

(iii) those that migrate cyclically and longitudinally, for purposes of procreation and to search for less adverse survival conditions.
   - Fish in this species migrate upstream to the river during the dry season where some spawn. Adults having spawned upstream then return to the downstream floodplain for the wet season, their young following later. Species in this group are often referred to as 'whitefish'.

These patterns are depicted in Fig. 2.

With an understanding of the environment and migratory patterns of fish we turn our attention to a mathematical model for the migration and dispersal of fish through floodplains and rivers.

3. Mathematical formulation

We make use of the derivation of reaction-diffusion models found in [16], making appropriate adjustments for the current model. Let $\partial\Omega$ be the boundary enclosing the two-dimensional domain Ω. Following the laws of conservation, we require that the rate of change of the amount of material in Ω is equal to the rate of flow of material across $\partial\Omega$ into Ω, plus the material created in Ω. Thus

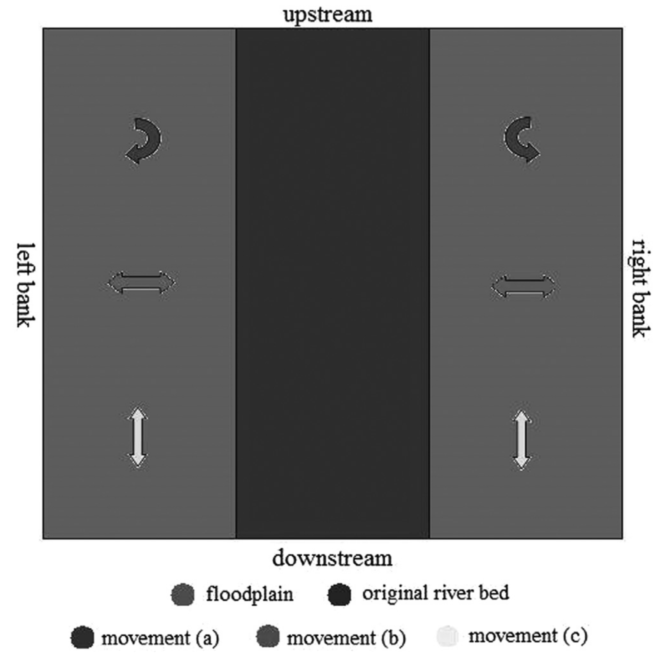

Fig. 2. Migratory patterns of fish populations being studied.

$$\frac{\partial}{\partial t}\int_{\Omega} u(\boldsymbol{x},t)\,d\boldsymbol{x} = -\int_{\partial\Omega} \boldsymbol{J}\cdot ds + \int_{\Omega} q(u,\boldsymbol{x},t)\,d\boldsymbol{x}, \tag{1}$$

where $u(\boldsymbol{x}, t)$ is the concentration of the fish species of interest, $\boldsymbol{J}$ is the flux of material, and $q(u, \boldsymbol{x}, t)$ is a net source of material (that is, $q > 0$ is an overall source and $q < 0$ is an overall sink). Applying the divergence theorem to the surface integral and assuming that $u(\boldsymbol{x}, t)$ is continuous in $\boldsymbol{x}$, (1) becomes

$$\int_{\Omega}\left[\frac{\partial u}{\partial t} + \nabla\cdot\boldsymbol{J} - q(u,\boldsymbol{x},t)\right]d\boldsymbol{x} = 0. \tag{2}$$

Since the domain Ω is arbitrary, the integrand must be zero and so the conservation equation for u is

$$\frac{\partial u}{\partial t} + \nabla\cdot\boldsymbol{J} = q(u,\boldsymbol{x},t). \tag{3}$$

This equation holds for a general flux transport $\boldsymbol{J}$, whether by diffusion or some other process. For this model we use a classical diffusion process, defining $\boldsymbol{J}$ as

$$\boldsymbol{J} = -\kappa\nabla u,$$

where κ is the diagonal matrix of positive diffusion coefficients (as in [13]). The minus sign indicates that diffusion transports matter from a high concentration to a low concentration. Thus (3) becomes

$$\frac{\partial u}{\partial t} - \nabla\cdot(\kappa\nabla u) = q(u,\boldsymbol{x},t). \tag{4}$$

Remark. The entries in the matrix κ can be constant, or they may depend on $\boldsymbol{x}$ and/or t.

The source term q will allow us to incorporate population decline, migratory effects, and population growth. While it was originally hypothesized that a Malthusian (exponential) growth model was most appropriate, one recognizes the fault in assuming that populations are able to grow at an exponential rate with no bound. Instead, we adopt a Verhulst (logistic) growth function as in [3]

$$a(\boldsymbol{x})u - \gamma(\boldsymbol{x})u^2 - \sigma(\boldsymbol{x})u,$$