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Passive synchronization for Markov jump genetic oscillator networks with time-varying delays

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ABSTRACT

In this paper, the synchronization problem of coupled Markov jump genetic oscillator networks with time-varying delays and external disturbances is investigated. By introducing the drive-response concept, a novel mode-dependent control scheme is proposed, which guarantees that the synchronization can be achieved. By applying the Lyapunov–Krasovskii functional method and stochastic analysis, sufficient conditions are established based on passivity theory in terms of linear matrix inequalities. A numerical example is provided to demonstrate the effectiveness of our theoretical results.

1. Introduction

The past decade has witnessed substantial progress toward the researches on genetic oscillator networks (GONs) due to their biological implications and potential applications in biological and biomedical science [1,2]. GONs are inherently coupled complex biochemical systems, where the nodes represent the genetic oscillators and the couplings denote the interactions between neighboring oscillators. With the development in mathematics and experiments concerning the genetic regulatory mechanisms, the dynamical behaviors of GONs are investigated to understand the living organisms. In particular, it has been demonstrated experimentally that the transition in genetic networks from one state to the next with certain transition probabilities can be determined by a homogeneous Markov chain with finite state. As a result, several important results of Markov jump genetic networks can be found in the remarkable papers [3–6].

Synchronization, as an emerging phenomenon of dynamically interacting systems, has played an important role in different fields, such as biology, ecology, sociology and technology [7–9]. Recently, some researchers have sparked an interest in the synchronization of GONs to offer insight into the nature of diverse biological systems and to design coupled GONs in practice [10–13]. It is worth mentioning that the external disturbances of GONs are always inevitable. Moreover, the time delays are particularly important for GONs due to the slow processes of transcription, translation, and translocation, such that it is necessary to address the delay effects in the mathematical models [14,15].

On the other hand, by utilizing an input–output description based on energy-related considerations, the passivity theory can provide an important framework for the analysis and design of control systems. The key idea behind this is that a large number of physical systems have certain input–output properties related to the conservation, dissipation and transport of energy. As is well known, a passive system having a positive definite storage function is Lyapunov stable. For such reasons, passivity theory has many applications in various areas, such as electrical circuits, nonlinear systems and complex networks [16–18]. Recently, the concept of passivity has attracted increasing attention in synchronization of chaos system and neural networks [19–23]. However, to the best of the authors' knowledge, for the passive synchronization of GONs, there is no result in the literature so far, which still remains open and challenging. This motivates us for this study.

The aim of this paper is to make one of the first attempts to investigate the synchronization problem of Markov jump GONs with time-varying delays and external disturbances. The regulation functions are assumed to be sector-like and the time-varying delays are bounded. The $1+N$ drive-response model is introduced for the synchronization of Markov jump GONs, which is different from the existing studies. By applying the Lyapunov–Krasovskii functional method and stochastic analysis, a model-dependent passivity-based strategy

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is developed for achieving the stochastic synchronization of Markov jump GONs. In contrast with existing results on synchronization of GONs, the main contributions of this paper are as follows. First, the $1+N$ drive-response model is introduced, which is more applicable for the synchronization of GONs. Second, the controller design scheme is given to guarantee that the Markov jump GONs are synchronized, while on some existing work, the controller design problem was not considered. Third, passivity theory is used for the synchronization of GONs by which the disturbance attenuation can be obtained.

The rest of this paper is organized as follows. Section 2 introduces some preliminaries and formulates the synchronization problem. In Section 3, by utilizing the passivity theory, delay-dependent results on synchronization of Markov jump GONs are presented. Section 4 gives a numerical example to illustrate the effectiveness of the derived results and the conclusion is drawn in Section 5.

Notation: The notation used in this paper is fairly standard. $\mathbb{R}^n$ denotes the n dimensional Euclidean space, $\mathbb{R}^{m\times n}$ represents the set of all $m\times n$ real matrices and $\mathbb{N}^+$ stands for the set of positive integers. I and 0 represent identity matrix and zero matrix with appropriate dimensions, respectively. $\mathbb{L}_2[0,\infty)$ is the space of square-integrable vector functions over $[0,\infty)$. The notation $P>0$ means P is real symmetric and positive definite. The Kronecker product of matrices $Q\in\mathbb{R}^{m\times n}$ and $R\in\mathbb{R}^{p\times q}$ is a matrix in $\mathbb{R}^{mp\times nq}$ and denoted as $Q\otimes R$. In addition, in symmetric block matrices, * is used as an ellipsis for the terms that are introduced by symmetry and diag$\{\cdots\}$ denotes a block-diagonal matrix. For the notation $(\Omega,\mathcal{F},\mathcal{P})$, Ω represents the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. $E\{\cdot\}$ stands for the mathematical expectation. Finally, all matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Problem formulation and preliminaries

Consider a group of N coupled Markov jump GONs with time-varying delays defined in a complete probability $(\Omega,\mathcal{F},\mathcal{P})$, which can be described as

$$\dot{x}_i(t)=A(r_t)x_i(t)+B(r_t)f(x_i(t))+C(r_t)g(x_i(t-\tau(t)))+\sum_{1\le i<j\le N}G_{ij}\Gamma(r_t)x_j(t-\sigma(t)),\tag{1}$$

with each isolated node given by

$$\dot{x}_i(t)=A(r_t)x_i(t)+B(r_t)f(x_i(t))+C(r_t)g(x_i(t-\tau(t))).\tag{2}$$

$x_i(t)=[x_{i,1}(t),x_{i,2}(t),\dots,x_{i,n}(t)]^T\in\mathbb{R}^n$ is the state vector of the ith genetic oscillator representing the concentrations of proteins, mRNAs and chemical complexes. $f(x_i(t))=[f_1(x_1(t)),f_2(x_2(t)),\dots,f_n(x_n(t))]^T$ and $g(x_i(t-\tau(t)))=[g_1(x_1(t-\tau)),g_2(x_2(t-\tau)),\dots,g_n(x_n(t-\tau))]^T$ are monotonic functions satisfying the sector-bounded conditions, which usually are of the Michaelis–Menten or Hill form. $\Gamma(r_t)$ is the inner linking matrix in each node, respectively. $G=(G_{ij})_{N\times N}$ is the coupling matrix of the networks representing the coupling strength and topological structure of the networks. It is further defined as follows: if there is a link from oscillator j to oscillator i $(i\neq j)$ then $G_{ij}>0$; otherwise, $G_{ij}=0$, $G_{ii}=-\sum_{j=1,j\neq i}^{N}G_{ij}$; $\tau(t)$ and $\sigma(t)$ are time delays. All $A(r_t)$, $B(r_t)$, $C(r_t)$, $\Gamma(r_t)$ are known constant matrices with appropriate dimensions for a fixed system mode.

The initial condition associated with (1) is given as follows:

$$x_i(t)=\phi(t),\ r_t|_{t=0}=r_0\in\mathcal{I},\ t\in\max[\tau,\sigma],\ i=1,\dots,N.$$

Remark 1. As discussed in Section 1, the genetic oscillators are tightly coupled with each other, which exhibit the characteristics of the GONs. Note that the links from one oscillator to others are considered to be with time delays, which are more general and realistic when modeling GONs from the practice point of view.

The parameter r_t $(t\ge 0)$ represents a right-continuous Markov process on a complete probability space $(\Omega,\mathcal{F},\mathcal{P})$, taking values in a finite set $\mathcal{I}\triangleq\{1,\dots,M\}$ with generator $\Theta=\{\pi_{lm}\},\quad\forall l,m\in\mathcal{I}$ described as

$$\Pr(r_{t+\Delta t}=m:r_t=l)=\begin{cases}\pi_{lm}\Delta t+o(\Delta t) & \text{if } l\neq m,\\ 1+\pi_{ll}\Delta t+o(\Delta t) & \text{if } l=m,\end{cases}$$

with $\Delta t>0$, $\lim(o(\Delta t)/\Delta t)=0$ and $\pi_{lm}\ge 0$ $(l,m\in\mathcal{I},\ m\neq l)$ is the transition rate from mode l at time t to mode m at time $t+\Delta t$, while $\pi_{ll}=-\sum_{m=1,m\neq l}^{M}\pi_{ij}$ for $\forall l\in\mathcal{I}$. For convenience, in the sequel, each possible value of r_t is denoted by l, $l\in\mathcal{I}$.

The $1+N$ synchronization problem of the Markov jump GONs (1) is considered by using the drive-response configuration. Suppose that another isolated node out of the N coupled GONs (1) is selected as a drive system. According to the drive-response concept, the controlled response GONs are given by

$$\dot{y}_i(t)=A_ly_i(t)+B_lf(y_i(t))+C_lg(y_i(t-\tau(t)))+\sum_{1\le i<j\le N}G_{ij}\Gamma_ly_j(t-\sigma(t))+u_{li}(t)+F_l\varpi_i(t),\tag{3}$$

where $y_i(t)$ is the state vector of the response GONs, $u_{li}(t)$ and $\varpi_i(t)\in\mathbb{L}_2[0,\infty)$ denote the control input and the external disturbance, respectively.

Remark 2. In this $1+N$ drive-response synchronization model, when $N=1$, the model can be reduced to a normal drive-response synchronization model. Moreover, the model is capable of coupled synchronization model while the drive system is not considered.

Define the synchronization error $e_i(t)=y_i(t)-x_s(t)$, where $x_s(t)$ is the state vector of the drive genetic oscillator. Then the following synchronization error system can be obtained from (2) and (3):

$$\begin{aligned}e_i(t)=&A_le_i(t)+B_l(f(y_i(t))-f(x_s(t)))+C_l(g(y_i(t-\tau(t)))\\&-g(x_s(t-\tau(t))))+\sum_{1\le i<j\le N}G_{ij}\Gamma_ly_j(t-\sigma(t))\\&+u_{li}(t)+F_l(\varpi_i(t)-\varpi_s(t)).\end{aligned}\tag{4}$$

By using the matrix Kronecker product, the error dynamical networks (4) can be rewritten as

$$\begin{aligned}\dot{e}(t)=&(I_N\otimes A_l)e(t)+(I_N\otimes B_l)(\mathbf{f}(\bar{y}(t)))+(I_N\otimes C_l)(\mathbf{g}(\bar{y}(t-\tau(t))))\\&+(G\otimes\Gamma_l)y(t-\sigma(t))+u_l+(I_N\otimes F_l)\varpi(t).\end{aligned}\tag{5}$$

where

$$\begin{aligned}e(t)&=\left[e_1^T(t),\dots,e_N^T(t)\right]^T,\\ \mathbf{f}(\bar{y}(t)))&=\left[f^T(y_1(t))-f^T(x_s(t)),\dots,f^T(y_N(t))-f^T(x_s(t))\right]^T,\\ \mathbf{g}(\bar{y}(t-\tau(t)))&=\Big[g^T(y_1(t-\tau(t)))-g^T(x_s(t-\tau(t))),\dots,\\&\qquad g^T(y_N(t-\tau(t)))-g^T(x_s(t-\tau(t)))\Big]^T\\ y(t-\sigma(t))&=\left[y_1^T(t-\sigma(t)),\dots,y_N^T(t-\sigma(t))\right]^T,\\ u_l(t)&=\left[u_{l1}^T(t),\dots,u_{lN}^T(t)\right]^T,\\ \varpi(t)&=\left[\varpi_1^T(t)-\varpi_s^T(t),\dots,\varpi_N^T(t)-\varpi_s^T(t)\right]^T.\end{aligned}$$

For convenience, the output of the above error dynamical networks is given as $Z(t)=e(t)$. Moreover, the following assumptions are given.

Assumption 1. *The nonlinear functions $f(\cdot)$ and $g(\cdot)$ satisfy the following sector-like conditions [24,25]:*

$$0\le\frac{f_i(s)-f_i(t)}{s-t}\le\kappa_{1i},\quad 0\le\frac{g_i(s)-g_i(t)}{s-t}\le\kappa_{2i},\quad i=1,\dots,N.\tag{6}$$

Remark 3. By Assumption 1, it can be obtained that

$$f^T(y)(f(y)-\kappa_1y)\le 0,\quad g^T(y)(g(y)-\kappa_2y)\le 0\tag{7}$$