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Concentration distribution around a growing gas bubble in tissue

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ABSTRACT

This paper presents the concentration distribution around a growing nitrogen gas bubble in the blood and other tissues of divers who surface too quickly, when the ambient pressure through the decompression process is variable and constant. This effort is a modification of Sirinivasan et al. model (1999) [9]. The mathematical model is solved analytically to find the growth rate of a gas bubble in a tissue after decompression in the ambient pressure. Moreover, the concentration distribution around the growing bubble is introduced. The growth process is affected by ascent rate $\dot{\alpha}(t)$, tissue diffusivity D_T, initial concentration difference ΔC_0, surface tension σ and void fraction ϕ_0.

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1. Introduction

Decompression sickness (DCS) is a dangerous and occasionally lethal condition caused by nitrogen bubbles that form in the blood and other tissues of divers who surface too quickly or people who flight for long distances from the earth (Fig. 2).

Excess dissolved gas in tissues or blood can form gas bubbles after a change of ambient pressure, such as occurs when divers ascend from underwater or when aviators or astronauts are exposed to low-pressure environments; it is generally agreed that such bubbles are either the primary cause or the precipitating factor in decompression sickness (DCS) [11].

More information about DCS pain and target organs of bubble created during decompression stress can be found in Ref. [1].

The tendency for gases to leave solution and enlarge a seed bubble can be expressed by the following equation [1]

$$\Delta P = \sum(P_i) - P_{ab}, \tag{1}$$

where ΔP is the differential pressure or tendency for the gas to leave the liquid phase, $\sum(P_i)$ is the total tension of the gas in the medium and P_{ab} is the absolute pressure (i.e. the total pressure on the body plus the hydrostatic pressure), that is, for the bubble phase to be generated $\Delta P > 0$ which can be used to predict the required time for a tissue, at a given depth, to have a critical gas tension (supersaturation) that may generate gas bubbles after decompression.

Bubble dynamics models suitable for these applications assume the bubble to be either contained in an unstirred tissue (two-region model) or surrounded by a boundary layer within a well-stirred tissue (three-region model) [9]. Recently, some models have been used in probabilistic treatments of the occurrence of decompression sickness (DCS) in humans [10].

Srinivasan et al. [9] have shown that, the three-region model is suitable for the finite tissue. It is also shown that the two-region model is applicable only to bubble evolution in tissues of infinite extent and cannot be readily applied to bubble evolution in finite tissue volumes to simulate how such evolution is influenced by interactions among multiple bubbles in a given tissue.

More literature about previous models that describing the growth of gas bubbles in tissues and blood can be shown in [9], another good comparative study of 10 models of DCS is presented by Wienke [12]. His study presented a quantitative summary of computer models in diving applications.

Srinivasan et al. [8,9] have solved the problem in the case of quasi-static pressure. In this work, we have solved the problem in more general case, at which the effect of changing in concentration with the time takes place.

2. Analysis

The effort is devoted to study the three-region model (gas bubble, thin boundary layer and well-stirred finite tissue). For simplicity, we will neglect solvent vapor pressure and consider cases involving only a single diffusible gas.

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Nomenclature

A_1, A and $\widetilde{A}$	time dependent variables defined by Eqs. (16), (17) and (34), respectively
C	concentration of dissolved gas [mol m^{-3}]
C_∞	concentration of dissolved gas in the tissue far from the bubble [mol m^{-3}]
ΔC_0	$= C_\infty - C_0$, the concentration difference [mol m^{-3}]
D_T	gas diffusion coefficient in tissue [m^2 s^{-1}]
P_a	gas partial pressure in arterial blood, defined by Eq. (2) [N m^{-2}]
P_{amb}	ambient pressure [N m^{-2}]
P_{atm}	atmospheric pressure [N m^{-2}]
P_g	pressure of the bubble wall [N m^{-2}]
$\dot{Q}$	blood flow per unit tissue volume [s^{-1}]
$\Re$	general gas const. [N m mol^{-1} K^{-1}]
r	the distance from the origin of the bubble [m]
R_0	initial bubble wall radius [m]
R	instantaneous bubble wall radius [m]
$\dot{R}$	instantaneous bubble wall velocity [m s^{-1}]
t	time elapsed [s]
T	temperature of the gas inside the bubble [K]
x	dummy distance variable [m]
y	$= x/R$, dummy and dimensionless distance variable

Greek symbols

$\dot{\alpha}$	ascent rate [N m^{-2} s^{-1}]
α_b	gas solubility in blood [mol N^{-1}m^{-1}]
α_t	gas solubility in tissue [mol N^{-1}m^{-1}]
μ	constant given by Eq. (26)
σ	the surface tension of liquid surrounding the bubble [N m^{-1}]

Subscripts

0	initial-value quantities
c	after the decompression process (constant ambient pressure)
d	throughout the decompression process (variable ambient pressure)
g	constants and variables corresponding to the gas bubble
m	final or maximum value
T	constants and variables corresponding to the tissue

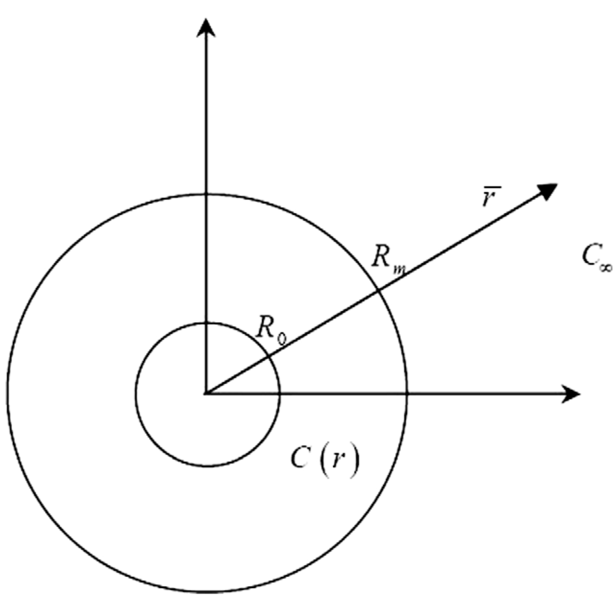

Fig. 1. The problem sketch.

A single gas bubble is considered to grow inside a tissue between two finite boundaries R_0 and R_m. The growth is affected by some parameters such as the pressure difference between the bubble pressure $P_g(R(t), t)$ and the ambient pressure $P_{amb}(t)$, surface tension of the mixture inside the tissue at the bubble boundary, concentration difference between the two phases and other physical parameters (Fig. 1).

The growth of the gas bubble has been studied based on the following assumptions:

- Gases are considered to be ideal.
- The bubble is assumed to have a spherical geometry.
- Pressure inside the bubble is assumed to be uniform.
- Gas density distribution inside the bubble is assumed to be uniform except for a thin boundary layer near the bubble wall.
- The viscosity of the fluid is omitted.

The mathematical model describing the current problem consists of four main equations (*mass, diffusion, Fick's and Laplace equations*).

1. *Mass balance equation*

Assuming equilibration of tissue gas with venous blood gas. The rate of gas uptake by the tissue is the amount carried by the blood

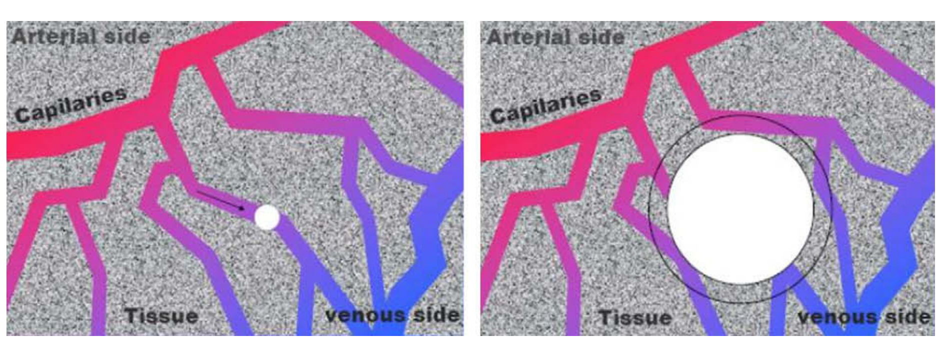

Fig. 2. On the left, in the initial phase of the decompression, an arterial bubble enters a tissue capillary net. It exchanges gas with the surrounding tissues and starts growing. If it reaches a critical radius, it might block the blood supply and cause ischemia. On the right, in the last phase of the decompression, a bubble has grown to a large volume using dissolved gas available in the surrounding tissue. Its mechanical action might cause pain [5].

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