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Intrinsic coherence resonance in excitable membrane patches

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Abstract

The influence of intrinsic channel noise on the spiking activity of excitable membrane patches is studied by use of a stochastic generalization of the Hodgkin–Huxley model. Internal noise stemming from the stochastic dynamics of individual ion channels does affect the electric properties of the cell-membrane patches. There exists an optimal size of the membrane patch for which the internal noise alone can cause a nearly regular spontaneous generation of action potentials. We consider the influence of intrinsic channel noise in presence of a constant and an oscillatory current driving for both, the mean interspike interval and the phenomenon of coherence resonance for neuronal spiking. Given small membrane patches, implying that channel noise dominates the excitable dynamics, we find the phenomenon of *intrinsic coherence resonance*. In this case, the relatively regular spiking behavior becomes essentially independent of an applied stimulus. We observed, however, the occurrence of a skipping of supra-threshold input events due to channel noise for intermediate patch sizes. This effect consequently reduces the overall coherence of the spiking.

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1. Introduction

A prominent question in neurophysiology concerns the limiting factors of the reliability of neuronal responses to given stimuli. In this work we focus on a particular aspect of this intricate issue, namely the impact of channel noise, which is generated by random gating dynamics of the ion channels in membrane patches of finite size. Since the work of Lecar and Nossal [1,2] it became clear, however, that not only the synaptic noise but also the randomness of the ion channel gating itself may cause threshold fluctuations in neurons [3]. Therefore, channel noise which stems from the stochastic nature of the ion channel dynamics must be taken into account [3]. It impacts such features as the threshold to spiking and the spiking rate itself [4–6], the anomalous noise-assisted enhancement of transduction of external signals, i.e., the phenomenon of Stochastic Resonance [7–12] and related, the efficiency for synchronization [13–15], to name but a few such interesting phenomena. The origin of the channel noise [3] is basically due to fluctuations of the mean number of open ion channels around the corresponding mean values. Therefore, the strength of the channel noise is mainly determined by the number of ion channels participating in the generation of action potentials.

In this paper we investigate within a stochastic generalization of the Hodgkin–Huxley model the influence of the channel noise on the mean interspike intervals and on the coherence of the spiking activity produced within the cell membrane containing a certain amount of ion channels. This is distinct from the effects of fluctuations due to synaptic noise which were studied within generalized Hodgkin–Huxley models [16,17]. The effect of coherence resonance describes a phenomena of self-synchronization of a system due to noise. It was observed for single excitable dynamics, e.g., for the Fitz Hugh–Nagumo model [18] or for a Hodgkin–Huxley model taking into account the influence of synaptic noise [16], and, as well as for neuronal networks [19,20].

2. The Hodgkin–Huxley model

Starting with the well-established model of Hodgkin and Huxley [21] we consider a cell-membrane patch of area $\mathscr{A}$ as an electrical capacitor possessing the specific area capacitance C. The membrane separates two ionic bath solutions (which in vivo correspond to the interior and the exterior of the excitable cell) with different concentrations of the ions of different sorts, mainly potassium, K^+, sodium, Na^+, and chloride, Cl^- ions. The macroscopic concentration differences are kept constant. In the cell this task is accomplished by ATP-driven ionic pumps. Furthermore, the ionic baths are on the average electrically neutral. However, due to the different ionic concentrations on the opposite sides of the semi-permeable membrane, the membrane becomes charged. As a consequence, an equilibrium transmembrane electrical potential difference emerges. The lipid membrane creates an almost impenetrable barrier for the ions. However, they can flow across the membrane through special ion selective pores created by specialized membrane proteins—the ion channels [22]. The specific potassium, I_K, and sodium, I_{Na}, ion currents through the open ion channels are approximately proportional to the differences of the transmembrane potential V and the specific (for the particular sort of ions) equilibrium potentials, E_K and E_{Na}, respectively. The *stochastically* averaged, mean conductances, $G_{Na}(m, h)$ and $G_K(n)$, are, however, strongly nonlinear functions of V. This nonlinearity emerges due to the gating dynamics (see below). There