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Mathematical Biosciences 201 (2006) 15–47

Mathematical Biosciences

www.elsevier.com/locate/mbs

Endemic threshold results in an age-duration-structured population model for HIV infection

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Received 23 June 2004; received in revised form 5 July 2005; accepted 3 December 2005
Available online 8 February 2006

Abstract

In this paper we consider an age-duration-structured population model for HIV infection in a homosexual community. First we investigate the invasion problem to establish the basic reproduction ratio R_0 for the HIV/AIDS epidemic by which we can state the threshold criteria: The disease can invade into the completely susceptible population if $R_0 > 1$, whereas it cannot if $R_0 < 1$. Subsequently, we examine existence and uniqueness of endemic steady states. We will show sufficient conditions for a backward or a forward bifurcation to occur when the basic reproduction ratio crosses unity. That is, in contrast with classical epidemic models, for our HIV model there could exist multiple endemic steady states even if R_0 is less than one. Finally, we show sufficient conditions for the local stability of the endemic steady states.

Keywords: HIV/AIDS epidemic; Structured population; Basic reproduction ratio; Threshold condition; Endemic state; Bifurcation

1. Introduction

During the past two decades, human immunodeficiency virus (HIV) disease has become one of the major public health problems in the world. For example, for many countries in Africa, AIDS has been already a major cause of death, it is predicted that it will soon become so in Asian

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0025-5564/$ - see front matter
doi:10.1016/j.mbs.2005.12.017

countries having a larger scale of populations. From theoretical point of view, the HIV/AIDS dynamics provides a large number of new problems to mathematicians, biologists and epidemiologists, since it has a lot of features different from traditional infectious diseases. Hence, the study of HIV/AIDS has stimulated the recent development of mathematical epidemiology. In the following we briefly discuss the characters which should be taken into account in mathematical models for the HIV dynamics.

First it is well known that HIV virus has the long incubation and infectious period. In the early stage of AIDS pandemic, its longest estimate was from 8 to 10 years, while now it could be prolonged by effective medical treatments. During the incubation period, the infectivity of infected people is varying depending on the time since infection. Thus, the time scale of HIV transmission is so long that demographic change of the host population could affect the transmission process. On the other hand, the death rate caused by AIDS is too high to be neglected, so the presence of HIV regulates the demographic structure of the host population. In summary, for HIV case we have to consider true interaction between demography and epidemics. This aspect has been often neglected in traditional epidemic models for common infectious diseases, since the time scale of the spread of such diseases is rather short in compare with the demographic time scale.

Next there exist various kind of risk groups for the HIV infection. HIV virus is transmitted by homosexual or heterosexual intercourse, needle sharing between drug abusers, blood transfusion, etc. Therefore, in the real, the susceptible population is composed of subgroups, each of which has a different susceptibility to the transmission of HIV virus. Even in a subgroup, individuals can be distinguished by the degree of risky behavior. Moreover, age-structure of the host population would play an important role, since social or sexual behavior of people heavily depends on their chronological age.

The whole dynamics of the spread of HIV/AIDS is so complex that we could not analyze it all at once. In this paper, we consider an age-duration-structured population model for the HIV infection in a homosexual community, while we neglect complexity which is caused by pair formation phenomena related to sex and persistence of unions. The reader interested in those aspects may refer to [10]. After the formulation of the basic system, we consider the initial invasion phase to calculate the basic reproduction ratio R_0, by which we can state the threshold criteria, that is, the disease can invade into the completely susceptible population if $R_0 > 1$, whereas it cannot if $R_0 < 1$. Next we consider the existence, uniqueness and bifurcation of endemic steady states. Finally, we examine the stability of endemic steady states.

2. The basic model

In the following, we consider an age-duration-structured population of homosexual men with a constant birth rate. For simplicity, individuals are assumed to be homogeneous with respect to their sexual activity, though the following argument could be easily extended to the risk-based model without any essential modification. Individuals have sexual contacts with each other at random and the duration of an exclusive partnership is negligibly short. We divide the homosexual population into three groups: S (uninfected but susceptible), I (HIV infected) and A (fully developed AIDS symptoms). We do not introduce a latent class, since the latent period of AIDS is negligibly short in compare with its long incubation period. Thus, it is assumed that