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On optimal spectrum-efficient routing in TDMA and FDMA multihop wireless networks ☆

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ABSTRACT

This paper addresses the problem of finding the route with maximum end-to-end spectral efficiency, under the constraint of equal bandwidth sharing, in multihop wireless networks that use time division multiple access (TDMA) or frequency division multiple access (FDMA). The conceptual difficulty of this problem arises from the fact that the associated routing metric is neither isotonic nor monotone, and, thus, it cannot be solved directly using shortest path algorithms. The author has recently presented the first polynomial-time algorithm that solves the problem to exact optimality for TDMA networks. The contribution of this paper is twofold. For TDMA networks, we present a new algorithm that achieves a significant improvement in the computational complexity as compared to the algorithms previously known. For FDMA networks, we introduce the first polynomial-time algorithm that provides *provably* optimal routes. The proposed algorithms rely on the divide-and-conquer principle and a modified Bellman–Ford algorithm for widest path computation. Our computational results further illustrate the efficiency of the proposed approach.

1. Introduction

Multihop wireless networks consist of a set of wireless devices that communicate with each other over multiple wireless hops, with intermediate nodes collaboratively routing ongoing traffic. The study of wireless multihop routing is a foundation for the development of emerging technologies such as [12]:

- *infrastructure wireless mesh networks*: wireless routers/access points are interconnected to provide an infrastructure/backbone for clients; or
- *WiMAX or IEEE 802.16*: a number of fixed relays can be used to extend the coverage area of a base station, and/or increase the capacity of a wireless access system, as in *IEEE 802.16j relay-based networks* [7].

The recent study in [2] has introduced the problem of finding the path with *maximum spectral efficiency* in multihop wireless networks that use time division multiple access (TDMA), under the constraint of equal bandwidth sharing. On the one hand, the authors of [2] note that simple shortest path algorithms cannot be used to solve the problem because the resulting routing metric is neither isotonic nor monotone [14]. On the other hand, exhaustive search has an exponential computational complexity because it involves pre-computing *all* paths joining a given node pair. Therefore, the study in [2] proposes two efficient, yet *sub-optimal* heuristics.

We have demonstrated in [12] that the spectrum-efficient routing problem for TDMA networks (originally introduced in [2]) can be, in fact, solved in polynomial-time. Our algorithm proposed in [12] is based on iteratively invoking (at most) M shortest path procedures, where M is the number of network links. This leads to an overall computational complexity of $O(N^2M)$ if the Dijkstra shortest path algorithm is used in each iteration, where N is the number of nodes. Since an N-node network has at most $N(N-1)$ directed links, the worst case complexity of the algorithm presented in [12] is $O(N^4)$.

Similarly to [2,12], this study combines tools from information theory and networking in an attempt to devise efficient spectrum-efficient routing algorithms that explicitly consider the impact of the physical layer. This is in contrast to studies from the information theory community (see, e.g., [10,17]) which focus on understanding the fundamental performance limits of the network, but lead to protocols that are typically too complex to be implemented in practical systems [2]. Our work contrasts also with studies from the networking community (see, e.g., [3,6]) which are often built on link-level abstractions of the network without fully considering the impact of the physical layer [2].

☆ A part of this paper was presented at IEEE Symposium on Computers and Communications (ISCC), June 2010 [13], and received a Best Paper Award.

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The contribution of this paper is twofold.

- For TDMA networks, we present a new algorithm that achieves a significant improvement in the computational complexity as compared to the algorithms previously known. In particular, we present an algorithm that provides *provably* optimal routes in $O(N^3)$ time.
- For networks that use frequency division multiple access (FDMA), we introduce the *first* polynomial-time algorithm that provides *provably* optimal solutions to the spectral-efficient routing problem. The computational complexity of this algorithm is also $O(N^3)$.

Our proposed approach relies on the divide-and-conquer principle, and the resulting algorithms (for FDMA and TDMA networks) are based on a *single* run of a modified Bellman–Ford algorithm for widest path computation.

The remainder of this paper is organized as follows. Section 2 provides a formal definition of the problem for TDMA networks. Section 3 provides the novel, improved polynomial-time algorithm for TDMA networks. The problem formulation, and polynomial-time algorithm for FDMA networks are presented in Section 4. Numerical examples and results will be presented in Section 5. Section 6 concludes the paper.

2. TDMA problem definition

A multihop wireless network is modeled as a graph $G = (V, E)$, where V represents the set of nodes (vertices) and E represents the set of links (edges). We let $l \in E$ signify a link in the network. We also let $N = |V|$ and $M = |E|$ denote the number of nodes and links, respectively.

Following [2,12], we consider the setting in which all transmit devices are constrained by the same symbol-wise average transmit power P, and assume that all devices transmit with power P when transmitting. A possible justification for this assumption is that nodes in *infrastructure* wireless mesh networks are mostly immobile and connected with abundant power supplies. Therefore, for a link $l \in E$, the signal-to-noise ratio (SNR) is given by:

$$SNR_l = \frac{PG_l}{N_0 B}, \quad (1)$$

where G_l is the path gain from the sender of link l to the receiver of link l, N_0 is the normalized one-sided power spectral density of the additive white Gaussian noise (at any receiver in the network), and B is the finite bandwidth of the wireless channel.

To avoid the difficulty (NP-hardness) of joint optimal routing and medium access control (MAC) layer scheduling, and following [2,12], it is assumed that a common channel is shared among all nodes using time division multiple access (TDMA) without spatial reuse. In other words, each node transmits in its own unique time slot, and uses the entire bandwidth (B) when transmitting. It is demonstrated in [2] that, even though a path is selected assuming no spatial reuse/interference, applying a scheduling technique (separately) that allows some spatial reuse to the selected path can further improve the spectral efficiency. Therefore, our framework can still benefit from spatial reuse. It is worth noticing that the MAC layer of the IEEE 802.16 mesh protocol, for example, is based on TDMA (see, e.g., [5]).

The spectral efficiency $R(L)$ of an arbitrary path L in the network is defined as the bandwidth-normalized end-to-end rate, i.e., $R(L) = C_L/B$ (in bps/Hz), where C_L is the end-to-end achievable data rate (in bps) and B is the channel bandwidth (in Hz) [2].[1] Under the constraint of equal bandwidth sharing, the TDMA achievable end-to-end data rate for path L can be expressed using the well-known Shannon capacity formula as (see, e.g., [4,16]):

$$C_L = \min_{l \in L} \frac{B}{|L|} \log\left(1 + \frac{PG_l}{N_0 B}\right), \quad (2)$$

where $|L|$ is the number of links (hops) in path L. Note that the factor $1/|L|$ results from the sharing of bandwidth equally among relay links, i.e., each link l along path L is allocated a time fraction $1/|L|$. Consequently, the end-to-end spectral efficiency ($R(L) = C_L/B$) of path L is given by [2]

$$R(L) = \min_{l \in L} \frac{1}{|L|} \log\left(1 + \frac{PG_l}{N_0 B}\right). \quad (3)$$

For a given path L, we define:

$$w(L) = \min_{l \in L} \log\left(1 + \frac{PG_l}{N_0 B}\right). \quad (4)$$

Note that $\log\left(1 + \frac{PG_l}{N_0 B}\right)$ can be viewed as the width of link $l \in E$. Therefore, $w(L)$ as defined by (4) is the width of the bottleneck link along path L, i.e., $w(L)$ is the width of path L. In the sequel, we use the terms link width and path width to refer to $\log\left(1 + \frac{PG_l}{N_0 B}\right)$ and $\min_{l \in L} \log\left(1 + \frac{PG_l}{N_0 B}\right)$, respectively.

By combining (3) and (4), the end-to-end spectral efficiency of any path L can be expressed as

$$R(L) = \frac{w(L)}{|L|}. \quad (5)$$

Note that the spectral efficiency of any path L, as given by (5), can be viewed as the ratio of the width of path L to its hop-count.

Given a source–destination (s–d) pair of nodes $(s, d) \in V \times V$, the problem of finding the route with maximum end-to-end spectral efficiency under the constraint of equal bandwidth sharing can, thus, be expressed as the following optimization problem:

$$\max_{L \in \mathcal{L}_{sd}} \frac{w(L)}{|L|}, \quad (6)$$

where $\mathcal{L}_{sd}$ is the set of all paths L connecting the source–destination pair of nodes $(s, d) \in V \times V$.

It was noted in [2] that Bellman–Ford or Dijkstra shortest path algorithms cannot be used to solve (6) directly because the routing metric $w(L)/|L|$ is neither isotonic nor monotone [14]. Problem (6), however, can be solved by an exhaustive search over all (possibly *exponentially* many) paths. This exponential complexity makes exhaustive search prohibitive in networks with a moderate-to-large number of nodes. The study in [2] presents two efficient algorithms that provide suboptimal solutions to (6). In [12], however, we have shown that *provably optimal* solutions to the problem can be obtained at an $O(N^4)$ worst-case complexity. In what follows, we present an improved algorithm with a computational complexity of $O(N^3)$, while still providing provably optimal solutions to the problem.

3. Optimal reduced-complexity algorithm

3.1. Main result

The main idea of the proposed algorithm is based on the following two facts:

[1] In other words, the spectral efficiency is defined as the rate at which data can be transmitted over a path per unit bandwidth. Hence, it is an indication of how efficient the channel bandwidth (which is a scarce resource) is utilized.