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Optimal policies aimed at stabilization of populations with logistic growth under human intervention

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ABSTRACT

This work examines both positive and negative impacts that economic growth may have on the ecological dynamics and stability of a single biological species. Local extinction of the species may force the social planner to implement defensive expenditures aimed at conservation of the species population by means of habitat protection. The latter may lead to an ecological equilibrium that will be different from the natural equilibrium that would have arisen in the absence of human intervention. Moreover, the existence of such equilibrium is formally demonstrated and its stability properties are revised. Additionally, optimal-choice decision policies are constructed on the basis of Pontryagin's maximum principle. Under such policies together with initial abundance of the species, the growth trajectories will move the system towards the fixed point of maximum species abundance.

1. Introduction

Historical evidence shows that nature evolution itself may lead to extinction of some species (for example, think of dinosaurs) or the latter may be caused by natural phenomena or disasters. On the other hand, almost all scientists acknowledge that the rate of species loss is greater now than at any time in human history. According to Ehrlich (1988), the primary cause of the decay of organic diversity is not direct human exploitation or malevolence, but the habitat destruction that inevitably results from the expansion of human population and human activities. Apart from the habitat loss, Armsworth et al. (2004) point out that climate change (in the long-term projection) also presents an increasingly important threat to extinction of particular species.

Apparently, habitat loss, climate change and increasing levels of pollution reflect together the negative side-effects of human intervention and aggregated production. Even without direct consumption of the species (harvesting), these factors may provoke drastic reductions of the species populations up to causing that some species become locally extinct.

In recent decades, there has been noted an increasing interest in research on relation and interaction between economic growth, sustainable development and natural environment conservation. A critical review done by Eppink and Van DenBergh (2007) summarizes key features of four basic categories of models that integrate economic theories and strategies aimed at species conservation. The majority of these models are designed in order to help a social planner to define strategies for optimal and/or sustainable harvesting, where species preservation guarantees the profit stability in the future and thus contributes to the economic development.

Some of them use optimal control techniques to study the stability properties of environmental and biological systems modified by human intervention. In particular, Li and Löfgren (1998) have developed an optimal control model for preservation of biodiversity using the richness of species as a diversity measure and also have examined the effect of interaction between human activity and biodiversity on the stability of the economic and ecological systems. However, this model includes the direct harvesting of the species (expressed by means of a control variable) and contains an exogenous parameter reflecting the effects of global environmental changes.

Alternatively, Swanson (1994) had proposed an amendment to classical harvesting models (see, e.g. Clark, 1976) by including another control variable that expresses the allocation of resources required for a species' survival. That can be treated as an initial attempt to modify the biological dynamics of species growth with defensive actions of policy-makers aimed at species protection.

Swanson's idea was further developed by Alexander (2000) who also proposed to include in the objective function both consumptive and *non-consumptive*[1] values of the biological species.

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[1] R. Alexander also cites several previous studies that addressed some non-consumptive values (principally, tourism values) of particular biological species; however, none of these studies had explicitly included such non-consumptive values in the mathematical models.

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This approach can be applied to analyze the population dynamics of endangered species that have both consumptive and non-consumptive values in order to design long-term the appropriate conservation policies for their sustainability and bioeconomic viability.

On the other hand, there are many wildlife species with no harvest value that are currently in threat by negative side-effects of human activity (urbanization, pollution, habitat loss, etc.). To study the evolution of such species, Alexander and Shields (2003) had proposed a non-harvesting variant of a dynamic model for one particular species (New Zealand's yellow-eyed penguin) using as control variable an index of the quantity of land resources which are vital for the species survival. The latter can be viewed as a defensive expenditure of the society aimed at the conservation of the natural habitat of the species. In fact, this non-harvesting model does not explicitly include the negative impact that human activity and aggregated production may have on the natural evolution of species population.

However, the results of Antoci et al. (2005a,b) clearly demonstrate that both negative and positive human actions may alter the stability properties of the natural dynamics of biological species. Their studies, performed on a basis of a dynamic model of two interacting species with linear dynamics, revealed an interesting fact. Namely, if the equilibrium level of the species is high enough, the local stability's properties will be preserved when the natural biological dynamics (without human intervention) is amended with economic and ecological features (that is, negative impact of aggregated production and positive impact of defensive expenditures). Additionally, Campo-Duarte and Vasilieva (2011) have come to the same conclusion using as a basis a single-species model with Gompertz population growth.

The present work intends to contribute to this strand of research by analyzing a non-harvesting model described by logistic equation and amended with economic and ecological features initially introduced by Antoci et al. (2005a,b). Besides confirming the general trend on preservation of local stability (claimed by Antoci et al., 2005a,b and Campo-Duarte and Vasilieva, 2011), we also address several questions that raise from the model analysis and validation, namely: What is the impact of human intervention on the stability and number of biological equilibria? Can the defensive expenditures mitigate the negative impact of aggregated production and to what extent? Are there proper defensive strategies to save a species from local extinction and under which additional conditions they are viable? How these strategies can be adjusted to possible variations in economic, ecological and biological situation?

To answer these questions, we formalize our study by introducing a stylized bioeconomic model in continuous time that includes two state variables – the population of a specie (biological component) and the capital (economical component). The model, formulated in Section 2, also contains two control variables – the consumption related to aggregated production with its negative effects (such as pollution, reduction of specie's habitat, etc.) and the generic defensive expenditures aimed at the conservation of a specie (positive effects). We do not consider the harvesting (i.e., direct consumption of a specie) and suppose that the population dynamics of a specie is altered only by the negative side-effects of aggregated production and by positive effects of defensive actions aimed at prevention of single species from local extinction.

Additionally, underlying features of logistic dynamics concede us a possibility to disclose and visualize some essential changes in stability properties of the "amended" ecological dynamics, that is, natural population dynamics modified by human intervention. Such changes principally depend on the problem parameters, including both control variables. This intuitive inference is then formalized in the subsequent sections.

We presume that there is a social planner acting in the economy who chooses the levels of consumption and investments in defensive expenditures so as to maximize the present discounted value of total utility subject to the physical capital accumulation dynamics and the amended ecological dynamics. The utility function considered in this paper favors consumption over the species conservation,[2] hence the biological species is threatened by local extinction.

Having defined the decision criterion, the model is then formulated in terms of optimal control (Section 3). The application of Pontryagin's maximum principle results in a four-dimensional optimality system for which a direct solution is regarded as hardly possible. Alternatively, we demonstrate the existence of fixed points of the optimality system (Section 4) and briefly revise their properties of local stability (Section 5). These theoretical results are applied then in Section 6 in order to illustrate the design of optimal-choice policies that are capable to move the system towards its fixed point with maximum species abundance. Finally, Section 7 provides some conclusions and closing remarks.

2. Bioeconomic model

In this section we will construct the bioeconomic model in continuous time and formulate the core problem in terms of optimal control.

For the sake of simplicity, consider a single-species unstructured population model given by Verhulst (or *logistic*) equation

$$\frac{dx}{dt} = rx(t)\left(1 - \frac{x(t)}{K}\right), \qquad x(t_0) = x_0 \tag{1}$$

where $x(t)$ describes the population size at time t (i.e., number of individuals, population abundance or biomass), $x_0 > 0$ is the initial size of population, constant $r > 0$ defines the growth rate of population and constant $K > 0$ is a carrying capacity parameter. The carrying capacity of a biological species in an environment is the maximum population size to be achieved in infinite time given the food, habitat, water and other necessities available in the environment. It is easily verified that (1) has two fixed points: $\underline{x}^* = 0$ (nodal repeller) and $\bar{x}^* = K$ (nodal attractor).

Let us suppose that species population obeys the evolutionary biological dynamics given by logistic equation (1) under the course of nature without human intervention. Eventually, the biological dynamics can be affected by side-effects of human activity such as aggregated production, pollution, urbanization, etc.

Let us introduce a simple growth model that links the biological dynamics (1) to a capital accumulation dynamics. For the sake of simplicity, we suppose that there is a single (global) good which is produced by capital alone:

$$\frac{dk}{dt} = k^{\alpha}(t) - c(t) - d(t) \tag{2}$$

where $k(t)$ is *per capita* capital and $k^{\alpha}(t)$ stands for *per capita* production function of Cobb–Douglas type with constant parameter α such that $0 < \alpha < 1$. Thus, the capital accumulation, described by Eq. (2), is used for aggregate production or reinvestment $k^{\alpha}(t)$, consumption $c(t)$ and defensive measures for environmental protection $d(t)$.

Remark 1. According to classical model of Solow (1956), $k = \mathcal{K}/\mathcal{L}$ where $\mathcal{K}$ is the capital stock and $\mathcal{L}$ is the labor input. The rate of capital increase (reinvestment) can be described by Cobb–Douglas production function

$$\frac{d\mathcal{K}}{dt} = \Pi(\mathcal{K}, \mathcal{L}) = \mathcal{K}^{\alpha}\mathcal{L}^{1-\alpha}.$$

[2] The same utility function was treated by Antoci et al. (2005a) and Campo-Duarte and Vasilieva (2011).