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A second-order impact model for forest fire regimes

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Abstract

We present a very simple "impact" model for the description of forest fires and show that it can mimic the known characteristics of wild fire regimes in savannas, boreal forests, and Mediterranean forests. Moreover, the distribution of burned biomasses in model generated fires resemble those of burned areas in numerous large forests around the world. The model has also the merits of being the first second-order model for forest fires and the first example of the use of impact models in the study of ecosystems.

Keywords: Forest fires; Savannas; Boreal forests; Mediterranean forests; Impact models; Chaos

1. Introduction

In savannas, as well as in boreal and in Mediterranean forests, wildfires are recurrent, but with remarkably different characteristics. Fires in savannas are almost periodic surface fires with return times ranging from 1 to 2 yr in moist areas (Goldammer, 1983) to 5–10 yr in arid areas (Rutherford, 1981). Fires in northern boreal forests are also quite regular, but they prevalently involve crowns (Kasischke et al., 1995) and occur every 50–200 yr (Rowe and Scotter, 1973; Zackrisson, 1977; Engelmark, 1984; Payette, 1989). By contrast, in Mediterranean areas, mixed (crown and surface) fires are almost the rule and occur in an apparently random fashion, with highly variable return times (Kruger, 1983; Davis and Burrows, 1994).

While it is true that natural forest fires originate from random events (mostly lightening) and are influenced by meteorological conditions (Bessie and Johnson, 1995), it is also true that fires can develop only if there is enough dry matter on the ground and if plants are sufficiently abundant in at least one of the various vegetational layers of the forest (for a relatively detailed discussion of this issue see Casagrandi and Rinaldi, 1999 and references therein). This suggests the idea that long-term predictions of forest fires can be roughly performed with deterministic models describing the growth processes, while more precise short-term predictions can only be performed through stochastic models (conceptually comparable with those used in weather forecast).

Here, we propose a simple deterministic model for the long-term prediction of forest fires in which the vegetational growth is described by standard ordinary differential equations, while fire episodes are modeled as instantaneous events. The fire develops when there is enough fuel on the ground, and, under suitable assumptions, this occurs when the mix of biomasses of the various layers reaches pre-specified values. The consequence of a fire is therefore an instantaneous reduction of the biomasses which is heuristically described by a simple rule in state space. Models with discontinuities of this kind are called "impact models" and have been first used in mechanics (see Brogliato, 1999 and references therein) to describe the dynamics of mechanical systems characterized by impacts among various masses. They are quite special and can be used to explain a number of rather subtle phenomena like the "Zeno chattering" (e.g., the diminishing return times of the impacts of a ping-pong bouncing ball) that other models cannot explain. Impact models represent the most naïve

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approach for the description of systems characterized by dynamic phenomena occurring at very diversified time scales (in our case the slow building up of biomass and its fast destruction through fire). They should not be confused with an apparently similar but substantially different class of models, namely that of periodically pulsed systems where the discontinuity in state space is generated by a periodic exogenous shock on the system. Many are the examples of this second class of models in biology: the control of continuously stirred-tank reactors (Funasaki and Kot, 1993), the study of pulsed chemotherapy (Lakmeche and Arino, 2001) and vaccination (Shulgin et al., 1998) and a number of contributions dealing with the effects of periodic harvesting or immigration (Ives et al., 2000; Liu and Chen, 2003; Chau, 2000; Grant et al., 1997; Geritz and Kisdi, 2004; Reluga, 2004). However, in those models the return time of the discontinuous event is constant and a priori fixed, while in our impact model the fire return times are neither constant nor pre-specified but are endogenously created by the interactions among the various layers of the forest.

The impact model we propose in this paper has only two differential equations (i.e. it is a so-called second-order model), one for the lower and one for the upper layer of the forest. We, therefore, exclude from our study single-layered shrublands. The simplest deterministic model available until now for the study of the dynamics of the fire return times was a standard (i.e. nonimpact) fourth-order model (Casagrandi and Rinaldi, 1999) in which the four state variables are the burning and nonburning biomasses of the lower and upper layers of the forest. It is important to keep in mind that the impact model we propose in this paper should not be intended as an approximation to that fourth-order model. However, it can mimic the qualitative features of the periodic fire regimes of savannas and boreal forest, as well as the chaotic fire regimes of Mediterranean forests suggested by the fourth-order model. As far as we know, this is also the first time that the general idea behind impact models is applied in ecology, although forestry and agricultural practices correspond very closely to the same idea: harvest when the population reaches a specified state. For this reason, it would be surprising if related models have not been applied in that context. In any case, there are certainly many other potential applications, since population dynamics are very often the result of slow dynamical processes interrupted by short devastating events.

2. The model

A continuous-time impact model is described by a set of n ordinary differential equations

$$\dot{x}(t) = f(x(t)), \tag{1}$$

which hold at any point in state space except on a $(n-1)$ dimensional manifold $\mathscr{X}^-$, where the impact occurs. When the state x reaches the manifold $\mathscr{X}^-$ at point x^-, an instantaneous transition described by a map

$$x^+ = \varphi(x^-) \quad x^- \in \mathscr{X}^- \tag{2}$$

occurs. The set

$$\mathscr{X}^+ = \varphi(\mathscr{X}^-)$$

is the set of the states of the system immediately after the impact. For this reason, the sets $\mathscr{X}^-$ and $\mathscr{X}^+$ are called, in general, *pre-* and *post-impact manifolds*. In the specific application considered in this paper, they simply represent the pre- and post-fire conditions of the forest and are therefore called *pre-* and *post-fire manifolds*. Obviously, first-order impact models are of no interest because if $n = 1$ the manifolds $\mathscr{X}^-$ and $\mathscr{X}^+$ are just two points and their most complex behavior is just a cycle passing through $\mathscr{X}^-$ and $\mathscr{X}^+$. This is why impact models are usually presented for $n \geqslant 2$.

The model we propose is a crude simplification of the real world. Species diversity, age structure, spatial heterogeneity, and plant physiology are not taken into account since we look only at total biomasses (see Shugart, 1984, Chapter 6 for a discussion). However, in order to distinguish fires in different layers, we assume that the forest is composed of two layers: a lower vegetational layer (from now on called "bush") that, depending on the forest, is composed of bryophytes, herbs, shrubs, or any mix of these plants, and an upper vegetational layer (from now on called "tree"), in general composed of plants of various species. The corresponding biomasses are denoted by B (bush) and T (tree). The equations of growth (1) characterizing our model are

$$\dot{B} = r_B B\left(1 - \frac{B}{K_B}\right) - \alpha BT,$$
$$\dot{T} = r_T T\left(1 - \frac{T}{K_T}\right). \tag{3}$$

This means that, in the absence of fire, trees grow logistically toward the carrying capacity K_T, while plants of the lower layer do not tend toward their carrying capacity K_B because tree canopy reduces light availability. A detailed discussion of the validity and limitations of Eqs. (3) can be found in Casagrandi and Rinaldi (1999), where realistic values for the five vegetational parameters $(r_B, r_T, K_B, K_T, \alpha)$ are also suggested.

As for the fire, we know (see, for example, Viegas, 1998) that the ignition phase is possible if there is enough dead biomass on the ground (leaves, twigs, branches, moss, herbs, etc.). Since the biochemical processes regulating the mineralization of dead biomass are relatively fast with respect to plant growth (Esser et al., 1982; Seastedt, 1988), we can reasonably assume, on the time-scale at which we describe bush and tree growth, that the rate of mineralization (proportional to the amount of dead biomass) equals the inflow rate of new necromass into the ground layer (proportional to bush and tree biomass). As a result, we can consider B and T as appropriate indicators of the abundance of fuel on the ground. Of course, also the water