Computer Networks 98 (2016) 1–13

Contents lists available at ScienceDirect

Computer Networks

journal homepage: www.elsevier.com/locate/comnet

A packet dropping mechanism for efficient operation of M/M/1 queues with selfish users☆

Yi Gai[a,*], Hua Liu[b], Bhaskar Krishnamachari[b]

[a] Intel Labs, Hillsboro, OR 97124, USA
[b] Ming Hsieh Department of Electrical Engineering, University of Southern California, Los Angeles, CA 90089, USA

ARTICLE INFO

Article history:
Received 13 December 2014
Revised 22 September 2015
Accepted 23 December 2015
Available online 28 January 2016

Keywords:
M/M/1 queue
Game theory
Price of Anarchy
Nash equilibrium

ABSTRACT

We consider a fundamental game theoretic problem concerning selfish users contributing packets to an M/M/1 queue. In this game, each user controls its own input rate so as to optimize a desired tradeoff between throughput and delay. We first show that the original game has an inefficient Nash Equilibrium (NE), with a Price of Anarchy (PoA) that scales linearly or worse in the number of users. In order to improve the outcome efficiency, we propose an easily implementable mechanism design whereby the server randomly drops packets with a probability that is a function of the total arrival rate. We show that this results in a modified M/M/1 queueing game that is an ordinal potential game with at least one NE. In particular, for a linear packet dropping function, which is similar to the Random Early Detection (RED) algorithm used in Internet Congestion Control, we prove that there is a unique NE. We also show that the simple best response dynamic converges to this unique equilibrium. Finally, for this scheme, we prove that the social welfare (expressed either as the summation of utilities of all players, or as the summation of the logarithm of utilities of all players) at the equilibrium point can be arbitrarily close to the social welfare at the global optimal point, i.e. the PoA can be made arbitrarily close to 1. We also study the impact of arrival rate estimation error on the PoA through simulations.

1. Introduction

In the past 20 years, the usage of the Internet has transitioned from being primarily academic/research-oriented to one that is primarily commercial in nature. In the current Internet environment, each commercial entity is inherently interested only in its own profit. Developing network mechanisms that are designed to handle selfish behavior has therefore gained increasing attention in recent years. The game theoretic approach, which was originally designed to model and guide decisions in economic markets, provides a valuable set of tools for dealing with selfish behavior [2–7].

In this work, we consider the network congestion problem at a single intermediate store-and-forwarding spot in the network. Several users send their packets to a single server with Poisson arrival rate. The server processes the packets on a first come first serve (FCFS) basis with an exponentially distributed service time. This is an M/M/1 queueing model [8]. There exists a tradeoff in this M/M/1 queueing model between throughput (representing the benefit from service), and delay (representing the waiting cost in the queue). In the gateway

☆ This research was sponsored in part by the U.S. Army Research Laboratory under the Network Science Collaborative Technology Alliance, Agreement number W911NF-09-2-0053, and by the U.S. National Science Foundation under CNS-0831545.This work is an extended version of the conference paper [1].

* Corresponding author. Tel.: +1 503 712 5198.
E-mail addresses: yigaiee@gmail.com, yi.gai@intel.com (Y. Gai), hual@usc.edu (H. Liu), bkrishna@usc.edu (B. Krishnamachari).

http://dx.doi.org/10.1016/j.comnet.2015.12.009
1389-1286/

congestion control context [9], a measure that is widely used to describe this trade-off is called "Power", which is defined as the weighted ratio of the throughput to the delay. When the users are selfish, we can formulate a basic M/M/1 queueing game. In this game, we assume that the users are selfish, and each control their own input arrival rate to the server. Each user's utility is modeled to be the power ratio for that user's packets.

This classic M/M/1 queueing game has been formulated and studied in [10–14]. The results from these prior works and our own results in this work are in agreement that the basic M/M/1 queuing game has an inefficient Nash Equilibrium. We are therefore motivated to design an incentive mechanism to force the users to operate at an equilibrium that is globally efficient. In particular, we focus on the design of a packet dropping scheme implemented at the server for this purpose. Our objective is that the dropping scheme should be as simple as possible, and it should minimize the Price of Anarchy (PoA, the ratio of the social optimum welfare to the welfare of the worst Nash equilibrium) to be as close to 1 as possible.

A key contribution of this work is the formulation of a modified M/M/1 queuing game with a randomized packet dropping policy at the server. We consider a simple and low overhead policy in our formulation, wherein the server need only monitor the sum of the rates of all users in the system. We show that this modified game with a packet dropping scheme is an *ordinal potential game* [15], which implies the existence of at least one pure Nash Equilibrium.

We show first that utilizing a step-function for packet dropping whereby the server drops all the packets when the sum-rate is greater than a threshold (and none when the sum-rate is below the threshold), results in infinite number of undesired Nash Equilibria which harms the PoA.

This raises the question whether a more sophisticated approach can do better. We show that indeed this is possible. In particular, we develop an incentive mechanism with a linear packet dropping that can improve the system efficiency to be arbitrarily close to the global optimal point (i.e., a PoA arbitrarily close to 1). This mechanism is similar to the Random Early Detection (RED) used for congestion avoidance on the Internet [16]. We prove the uniqueness of NE of the game with this mechanism. We also show that best response dynamics will converge to the unique NE.

Our paper is organized as follows. Section 2 summarizes the related work. We present the model of an M/M/1 queue game in Section 3. The social welfare and Price of Anarchy are described in Section 4 to investigate the efficiency of the NE. Then, in Section 5, we propose to design an incentive packet dropping scheme implemented at the server to improve the efficiency. Section 6 proves that the game defined with packet dropping policy is an ordinal potential game by giving the potential function. Section 7 shows the best response function. In Section 8, we show the behavior when utilizing a simple step-function for packet dropping. In Section 9 we propose the RED-like linear packet dropping incentive scheme. We show that with this scheme, it is possible to make the Price of Anarchy arbitrarily close to the optimal point. The uniqueness of NE of such a game is proved in Section 10. In Section 11, we show that the best response dynamics will converge to the unique Nash Equilibrium. In Section 12, we undertake simulations to see how the process of statistically estimating the input arrival rates in a real system would impact the PoA. We conclude the work in Section 13.

2. Related work

Throughput-delay tradeoffs in M/M/1 queues with selfish users have been previously studied in [10–14]. A utility function for each user is defined as the corresponding application's power and each user is treated as a player in such a game and adjusts its arrival rate to handle the trade-off between throughput and delay. Every user is assumed to be selfish and only wants to maximize its own utility function in a distributed manner.

Bharath-Kumar and Jaffe [10] wrote one of the earliest papers on the formulation of throughput-delay tradeoffs in M/M/1 queues with selfish users. The paper discusses the properties of power as a network performance objective function. A class of greedy algorithms where each user updates its sending rate synchronously to the best response of all other users' rates to maximize the power is proposed. Convergence of the best response to an equilibrium point is shown in this paper.

Douligeris and Mazumdar [11] extended Bharath-Kumar and Jaffe's work to the case with different weighting factors defined in the power function for different users and provided analytical results describing the Nash Equilibrium. They showed that the equilibrium point that the greedy best response dynamic algorithm converged to was a unique Nash Equilibrium.

The work by Zhang and Douligeris [12] proved the convergence of the best response dynamics for this basic M/M/1 queueing game under the multiple users case. Thus all these prior works [10–12] studied only variants of the basic game. Their work, along with ours, shows that this basic game results in an inefficient outcome. Our work is the first to develop a mechanism design for this problem that addresses this shortcoming by showing how to achieve near-optimal performance using a packet-dropping policy.

Dutta et al. [13] studied a related problem involving a server that employs an oblivious active queue management scheme, i.e. drops packets depending on the total queue occupancy with the same probability regardless of which flow they come from. They also consider an M/M/1 setting with users offering Poisson traffic to a server with exponential service time. The users' actions are the input rates and the utilities the goodput/output rates. The existence and the quality of symmetric Nash equilibria are studied for different packet dropping policies. Although our work also explores oblivious packet dropping schemes, it is different from and somewhat more challenging to analyze than [13], because our utility function reflects the tradeoff between goodput and delay.

In another, more recent work, [14], Su and van der Schaar have discussed linearly coupled communication games in which users' utilities are linearly impacted by their competitors' actions. An M/M/1 FCFS queuing game with the power as the utility function is one illustrative