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A framework to compare topology algorithms in multi-channel multi-radio wireless mesh networks

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A B S T R A C T

Creating an optimal topology for a multi-channel multi-radio wireless mesh network by an algorithm is a balancing act between different metrics such as bandwidth, delay, and redundancy. We propose a framework to quickly evaluate and compare resulting network graphs of different topology algorithms, both distributed and centralized, in wireless mesh networks. This framework complements to graph analysis and network simulation. The metrics presented in this paper, are both data flow (such as bandwidth capacity, delay, and loss) and structural characteristics (such as minimal edge and node cut) related. Each presented metric can be solved using a linear programming with the weighted incidence matrix of the network graph and the protocol interference model. The framework uses matrix operations, which are well established, making the framework unambiguous and easy to implement. We demonstrate the framework by comparing topology algorithms with increasing complexity and by comparing topology algorithms found in literature.

1. Introduction

Since wireless technology emerged in backbone networks as wireless mesh networks[1], classical network planning had to be revised. Elements such as frequencies, interference, noise, and bit-rates are additional variables to take into account during network planning. Yet, over time, those variables can change drastically or can be altered to improve the network.

A number of algorithms are designed to tackle the challenges arising from those alterations, and benefit from their improvements. A thorough overview of such algorithms can be found in [2–4]. The performance of an algorithm is shown by the designers via simulation results on a number of networks and for a number of metrics. For many topology algorithms [5–14] one of these optimized metrics is the node to node throughput in the network created by the algorithm. In [12] this metric is combined with a delay measurement between the nodes, in [11] combined with the fractional network interference and in [8] combined with both the delay and loss between the nodes. The simulation results validate for each algorithm its performance gain.

Although simulation results can be used during development of topology algorithms, simulating numerous networks with varying properties, takes time. We propose a framework enabling comparison of the resulting subgraphs of the topology algorithms using different metrics. The framework uses matrices for the calculations, which makes it unambiguous and easy to implement. We selected a number of metrics that can be divided in two groups. The first group has data flow characteristics including bandwidth capacity, delay, and loss, and the second group has network structure metrics including edge and node redundancy. Although these metrics already cover a variety of properties of the resulting network graphs, the metrics

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Table 1
Table of symbols.

Symbol	Description
$\mathcal{N}$	Set of nodes labelled n_i
$\mathcal{F}$	Set of interfaces labelled f_i
$\mathcal{E}$	Set of edges labelled e_i
$\mathcal{E}_L$	Topology selected edges, subset of $\mathcal{E}$
R_C	Communication range function
G	Communication graph $(\mathcal{F}, \mathcal{E})$
$\boldsymbol{N}$	$\lvert\mathcal{N}\rvert \times \lvert\mathcal{F}\rvert$ node matrix
$\boldsymbol{E}$	$\lvert\mathcal{F}\rvert \times \lvert\mathcal{E}\rvert$ incidence matrix of G
$\boldsymbol{W}$	$\lvert\mathcal{E}\rvert$ weight row vector
$\boldsymbol{H}$	$\lvert\mathcal{E}\rvert \times \lvert\mathcal{E}\rvert$ two-hop interference matrix
$\boldsymbol{L}$	$\lvert\mathcal{E}_L\rvert \times \lvert\mathcal{E}\rvert$ binding matrix
$\boldsymbol{E}_L$	$\lvert\mathcal{F}\rvert \times \lvert\mathcal{E}_L\rvert$ incidence matrix $\boldsymbol{EL}'$
$\boldsymbol{W}_L$	$\lvert\mathcal{E}_L\rvert$ weight row vector $\boldsymbol{WL}'$
$\boldsymbol{C}$	$\lvert\mathcal{E}_L\rvert \times \lvert\mathcal{E}_L\rvert$ channel matrix
$\boldsymbol{H}_{CW}$	$\lvert\mathcal{E}_L\rvert \times \lvert\mathcal{E}_L\rvert$ weighted channel interference matrix
$\boldsymbol{W}_{LC}$	$\lvert\mathcal{E}_L\rvert$ loaded interference weight row vector

presented are not exclusive and different metrics can be constructed and implemented within the framework, for example the number of runs the algorithm performs.

The framework proposed is complementary to graph analysis and simulation results of the network graph. Where graph analysis provides insight in the structure of the graph and the importance of nodes [15–19], it does not take interference or link rates into account. Simulations on the other hand, will evaluate each packet as it is transmitted from one interface to another. This results in a very detailed evaluation of a given scenario yet simulations are time consuming.

Our proposed framework stands between graph analysis and simulation by evaluating the network graph created by the algorithms using easy to calculate metrics that incorporate interference and edge or link weights. The interference model used in the proposed framework is the protocol interference model described in [20] and successfully applied in [21–25] and many others.

In Section 2 we define the different variables and notations used in the framework, summarized in Table 1. The evaluation metrics used in the framework are described in Section 3. Then, in Section 4 the steps performed by the model are explained: constructing random graphs, performing an optional channel allocation and handling the topology algorithms output. To demonstrate the capabilities of the model, different topology algorithms are presented in Section 5 and are evaluated in Section 6.

2. Notation and definitions

In this section we describe the used matrices and operations to define a network and the result returned by the topology algorithms. We use a matrix representation because this allows for easy implementation of the framework and unambiguous definitions.

2.1. Nodes and interfaces

A mesh network consists of wireless nodes. Such a node is denoted with n_i, with i the index of the node. To allow a node to communicate with other nodes, it needs at least one interface, called f_j with j the index of the interface. We define the finite set of all nodes of a mesh network $\mathcal{N} := \{n_1, n_2, \dots, n_{|\mathcal{N}|}\}$. Similar as $\mathcal{N}$, we define the finite set of all the interfaces of the mesh network as the set $\mathcal{F} := \{f_1, f_2, \dots, f_{|\mathcal{F}|}\}$.

Each node n_i of the mesh network is equipped with a number of wireless interfaces out of the set of interfaces $\mathcal{F}$. A mesh node n_i is defined as a set of interfaces, $n_i := \{f_j, f_{j+1}, \dots, f_{j+k}\}$. Interfaces within the same node can exchange data with a high rate, negligible to the lower rate between interfaces of different nodes. Therefore, communication between interfaces of the same nodes is not considered, yet interference between them is. The relationship between the nodes of $\mathcal{N}$ and the interfaces of $\mathcal{F}$ is defined in the $|\mathcal{N}| \times |\mathcal{F}|$ binary matrix $\boldsymbol{N}$. Each row represents a node and each column an interface.

$$\boldsymbol{N}_{ij} := \begin{cases} 1 & \text{if } f_j \in n_i \\ 0 & \text{else} \end{cases}$$

The element of a matrix $\boldsymbol{X}$ on the ith row and jth column is denoted by $\boldsymbol{X}_{ij}$. The row vector $\boldsymbol{N}_{i*}$ is the interface vector of node i. To indicate the ith row vector of a matrix $\boldsymbol{X}$, the notation $\boldsymbol{X}_{i*}$ is used where the $*$ denotes all the columns. Similarly, $\boldsymbol{X}_{*j}$ is the column vector of the jth column.

The structure of the matrix $\boldsymbol{N}$ is limited by the constraints that each interface is assigned to exactly one node $(\forall f_j \in \mathcal{F} : \sum_{i=1}^{|\mathcal{N}|} \boldsymbol{N}_{i,j} = 1)$ and each node has at least one interface $(\forall n_i \in \mathcal{N} : \sum_{j=1}^{|\mathcal{F}|} \boldsymbol{N}_{i,j} > 0)$.

2.2. Communication edges

Wireless interfaces can exchange information if they are within each other's communication range. Let the symmetric function on two variables $R_C : \mathcal{F} \times \mathcal{F} \to \{0, 1\}$ map a pair of interfaces to one if the two interfaces are within each other's communication range and to zero otherwise. The function is defined symmetric because information transmitted from interface f_i to f_j is acknowledged by f_j. This means that a transmission of a data packet is only successful if the data packet is successful transmitted from f_i to f_j and the data packet is successful acknowledged by f_j. Therefore, the communication between the two interfaces must be symmetric.

Although interfaces on the same node are most likely within each other's communication range, those interfaces can exchange information via other means and their communication relationship is excluded. Therefore we define the finite set $\mathcal{E}$ as the set of unordered pairs of interfaces not of the same node and within each other's communication range. Then $G := (\mathcal{F}, \mathcal{E})$ is the communication graph with $\mathcal{F}$ the vertex set and $\mathcal{E}$ the edge set.

$$\mathcal{E} := \left\{ \left\{ f_i, f_j \right\} \mid \nexists n \in \mathcal{N} : f_i, f_j \in n \text{ and } R_C\left(f_i, f_j\right) = 1 \right\}$$

The set $\mathcal{E}$ is the set of unordered pairs of interfaces that are not in the same node and within each other's communication range. Each pair is uniquely named e_i with $1 \le i \le |\mathcal{E}|$. The $|\mathcal{F}| \times |\mathcal{E}|$ matrix $\boldsymbol{E}$ is defined as the undirected incidence matrix of this communication graph G with the interfaces of $\mathcal{F}$ as rows and the edges $\{e_1, e_2, \dots, e_{|\mathcal{E}|}\}$ as