Computer Networks 58 (2014) 13–28

Contents lists available at ScienceDirect

Computer Networks

journal homepage: www.elsevier.com/locate/comnet

A branching process approach to compute the delay and energy efficiency of tree algorithms with free access

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ARTICLE INFO

Article history:
Received 8 February 2013
Received in revised form 1 July 2013
Accepted 14 August 2013
Available online 2 September 2013

Keywords:
Conflict resolution
Tree algorithms
Delay
Energy usage
Branching process

ABSTRACT

This paper presents a branching process approach to determine the main performance measures of a variety of conflict resolution algorithms known as tree algorithms with free access. In particular we present an efficient approach to calculate the mean delay, number of transmission attempts, collision resolution interval length and energy usage with arbitrary precision.

1. Introduction

Tree algorithms (TAs) form a well studied class of conflict resolution algorithms [1–7]. Depending on the manner in which new users join the channel, they are termed *free* or *blocked* access TAs. Under free access users transmit new packets at the start of the next time slot, meaning no channel sensing is required. Under blocked access some rules are in place that indicate when a new user is allowed to transmit a packet for the first time, which requires either limited or full sensing of the channel. It is fair to state that free access algorithms are typically harder to analyze than their blocked access counterparts, which is one of the reasons why fewer results on free access algorithms have appeared in the literature.

In this paper we extend the branching process approach of [8], which was used to study the maximum stable throughput (MST) of the same class of tree algorithms as considered in this paper, to analyze the mean delay, mean number of transmission attempts, mean length of the conflict resolution interval (CRI) and mean energy usage. As such the current paper heavily relies on the technique developed in [8] and the derivation of the mean length of the CRI is a rather trivial extension. However, extending the approach in [8] to determine the mean delay, mean number of transmission attempts and mean energy usage is far less obvious as one needs to determine, amongst others, the distribution of the number of packets that are transmitted during a packet's first transmission attempt (called the top-of-stack observed states in [9]). Once this distribution is obtained, it is not hard to adapt the branching process of [8] to determine the mean number of transmission attempts, but adapting it to determine the mean delay (and energy usage) from this distribution is still less obvious.

Existing results for the mean delay or energy characteristics [9–11] are typically expressed through some operator $S(f(\cdot),z)$ (see Section 4.6 for details), the numerical evaluation of which is very time and memory consuming and can often only be used to compute the first 4 or 5 digits accurately. Our branching approach on the other hand requires hardly any time (a fraction of a second) or memory and is able to produce results with very high accuracy (15 digits or more).

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http://dx.doi.org/10.1016/j.comnet.2013.08.022

One of the key characteristics of the branching process approach in [8] is the use of the truncation parameter d. Numerical experiments showed that the MST could be determined with a precision of as many as 15 digits even for moderate values of d, e.g., $d \leqslant 20$. The branching process approach introduced in this paper also relies on the truncation parameter d and we show that the approach can be used to calculate the performance measures of interest up to arbitrary precision by comparing them to the results presented in [9–12]. The method of [13,14] cannot be used to study the impact of d as it relies on a similar truncation. Apart from reproducing the existing results of [9,11,12], we also obtain many new results for other TAs with free access. All of these novel results were confirmed by means of discrete event simulations.

We consider the following standard channel model and user behavior (see [3–5] for a detailed discussion):

1. The channel is divided into fixed length time slots and all packets have the same length as a slot. Users are only allowed to start transmitting at the beginning of a time slot.
2. There is an infinite set of users whose aggregate packet generation process is a Poisson process with rate $\lambda > 0$.
3. Whether a transmission is successful only depends on the number of packets sent during the particular slot. Packet reception fails whenever two or more packets are transmitted simultaneously.
4. At the end of each time slot, the receiver sends either binary (collision/no collision) or ternary (idle/success/collision) feedback to the users.

While analyzing the TAs considered in this paper we often relax some of the four assumptions above, for instance we will consider channels with errors, multiple reception capabilities and probabilistic capture. Although all the presented numerical results assume Poisson arrivals, the branching process approach presented in this paper can be readily extended to any arrival process in which the number of arrivals in consecutive time slots are independent and identically distributed.

The paper is structured as follows. We start by discussing some related work in Section 2. Next, in Section 3 we briefly revisit the operation of the basic q-ary TA, the definition of a multi-type branching process and the manner in which these processes were used in [8] to determine the MST. Our branching process approach to compute the mean delay, mean number of transmission attempts, mean CRI duration and mean energy usage of the basic q-ary TA is discussed in detail in Section 4, while Section 5 indicates that the same general approach can be used to analyze a large variety of free access TAs without much additional effort. All the results in Sections 4 and 5 have either been validated by existing results or simulation. Concluding remarks are given in Section 6.

2. Related work

The maximum stable throughput (MST) under free access was obtained for the basic and modified q-ary TA in [15,2], for a channel with errors in [16,17], for variable length packets in [10], while [12] analyzed the impact of having some control sub-channels with separate feedback and [18] considered a system in which an interference cancellation mechanism is deployed. For other performance measures such as the delay or energy usage even fewer analytical results are available.

The delay of the basic binary TA algorithm with free access is analyzed in [9] using functional equations. The solution of these equations is expressed through some operator $S(f(\cdot),z)$ which is defined as a sum over a semigroup H. Numerically evaluating this operator (for $p \neq 1/2$) is computationally heavy and considerable care is needed even when computing the first 5 digits only. A similar approach as in [9] was used in [10] to obtain the delay characteristics of a TA with variable length packets. The mean energy usage of the basic binary TA with free access was determined in [11] and was defined based on the mean delay $E[D]$ and mean number of transmission attempts $E[T]$, where $E[T]$ was expressed using the same operator $S(f(\cdot),z)$. A computational method based on matrix analytic methods to calculate the mean delay of the basic q-ary TA was also presented in [13,14]. Finally, [12] provided a closed form expression for the mean delay in case of an infinite number of control sub-channels, which corresponds to the coordinated splitting algorithm in [8].

More recently, a novel technique to determine the MST of TAs with free access was presented in [8]. Given the arrival rate λ and a particular TA with free access, the technique existed in defining a branching process such that the process is sub-critical if and only if the TA is stable under Poisson arrivals with rate λ. The MST of various TAs with free access could therefore be determined in an efficient manner by means of a simple bisection algorithm.

3. Preliminaries

Before introducing our branching process approach in Section 4, we briefly discuss the operation of the basic q-ary algorithm, the definition of a multi-type branching process and the branching process used in [8] to determine the MST of the basic q-ary TA.

The basic q-ary TA with free access, where $q \geqslant 2$ is an integer, operates in the following manner. Whenever a user has a packet ready for transmission, he becomes active. All active users maintain a single variable called the *counter*, the value of which is updated at the end of each time slot. A user is allowed to transmit in the next slot whenever his counter equals zero. Users that become active initialize their counter to zero and are therefore allowed to transmit in the next time slot. Binary feedback is provided at the end of each time slot and the counter of a user is updated as follows:

1. If the slot holds a collision, all users with a counter larger than zero increase their counter by $q-1$. Each user involved in the collision sets his counter equal to i, with probability p_{i+1}, for $i = 0, \ldots, q-1$. The probabilities $p_1, \ldots, p_q$ are protocol parameters that sum to one.