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Continuum equilibria and global optimization for routing in dense static ad hoc networks

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ABSTRACT

We consider massively dense ad hoc networks and study their continuum limits as the node density increases and as the graph providing the available routes becomes a continuous area with location and congestion dependent costs. We study both the global optimal solution as well as the non-cooperative routing problem among a large population of users where each user seeks a path from its origin to its destination so as to minimize its individual cost. Finally, we seek for a (continuum version of the) Wardrop equilibrium. We first show how to derive meaningful cost models as a function of the scaling properties of the capacity of the network and of the density of nodes. We present various solution methodologies for the problem: (1) the viscosity solution of the Hamilton–Jacobi–Bellman equation, for the global optimization problem, (2) a method based on Green's Theorem for the least cost problem of an individual, and (3) a solution of the Wardrop equilibrium problem using a transformation into an equivalent global optimization problem.

1. Introduction

Research on ad hoc networks involves the design of protocols at various network layers (MAC, transport, etc.), the investigation of physical limits of transfer rates, the optimal design of end-to-end routing, efficient energy management, connectivity and coverage issues, performance analysis of delays, loss rates, etc. The study of these issues has required the use of both engineering methodologies as well as information theoretical ones, control theoretical tools, queueing theory, and others. One of the most challenging problems in the performance analysis and in the control of ad hoc networks has been routing in massively dense ad hoc networks. On one hand, when applying existing tools for optimal routing, the complexity makes the solution intractable as the number of nodes becomes very large. On the other hand, it has been observed that as an ad hoc network becomes "more dense" (in a sense that will be defined precisely later), the optimal routes seem to converge to some limit curves. This is illustrated in Fig. 1. We call this regime, the limiting "macroscopic" regime. We shall show that the solution to the macroscopic behavior (i.e., the limit of the optimal routes as the system becomes more and more dense) is sometimes much easier to solve than the original "microscopic model".

The term "massively dense" ad hoc networks is used to indicate not only that the number of nodes is large, but also that the network is highly connected. By the term "dense" we further understand that for every point in the plane there is a node close to it with high probability; by "close" we mean that its distance is much smaller than the transmission range. In this paper and in previous works (cited in the next paragraphs) one actually studies the limiting properties of massively dense ad hoc networks, as the density of nodes tends to infinity.

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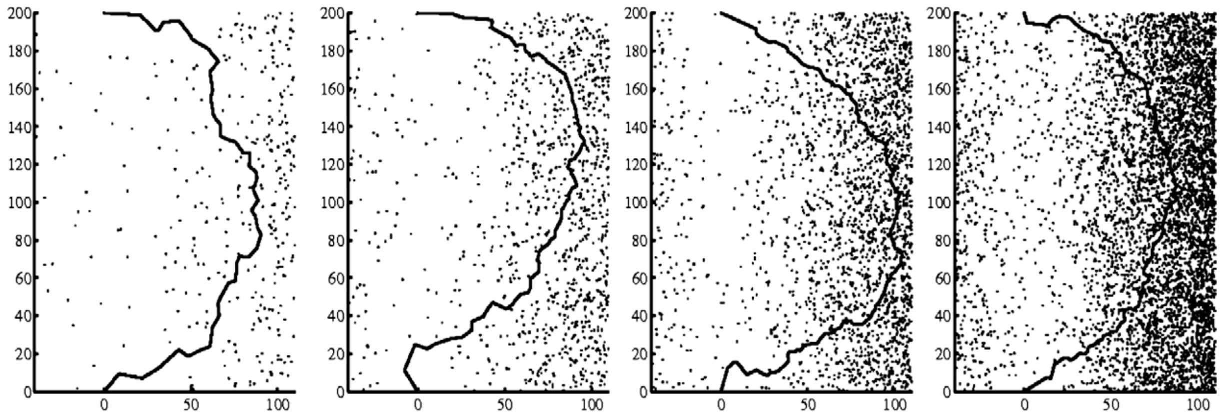

Fig. 1. Minimum cost routes in increasingly large networks.

The empirical discovery of the macroscopic limits motivated a large number of researchers to investigate continuum-type limits of the routing problem. A very basic problem in doing so has been to identify the most appropriate scientific context for modelling and solving this continuum limit routing problem. Our major contribution is to identify completely the main paradigms (from optimal control as well as from road traffic engineering) for the modelling and the solution of this problem. We illustrate the use of these methodologies by considering new types of models that arise in the case of nodes with directional antennas.

1.1. Physics-inspired paradigms

The physics-inspired paradigms used for the study of large ad hoc networks go way beyond those related to statistical-mechanics in which macroscopic properties are derived from microscopic structure. Starting from the pioneering work by Jacquet (see [18]) in that area, a number of research groups have worked on massively dense ad hoc networks using tools from geometrical optics [18].[1] Popa et al. in [22] studied optical paths and actually showed that the optimal solution to a minmax problem of load balancing can be achieved by using an appropriately chosen optical profile. The forwarding load appears to correspond to the scalar sum of traffic flows of different classes. This means that the optimal solution (with respect to this objective) can be achieved by single path routes, a result obtained also in [15]. Similar problems have been also studied in [8], as well as in works doing load balancing by analogies to Electrostatics (see e.g. [13,19,20,27,28], and the survey [29] and references therein). We shall describe these in the next sections.

The physical paradigms allow the authors to minimize various metrics related to the routing problem. In contrast, Hyytia and Virtamo proposed in [16] an approach based on load balancing arguing that if shortest path (or cost minimization) arguments were used, then some parts of the network would carry more traffic than others and may use more energy than others. This would result in a shorter lifetime of the network since some parts would be out of energy earlier than others.

1.2. Road-traffic paradigms

The development of the original theory of routing in massively dense networks among the community of ad hoc networks has emerged in a complete independent way of the existing theory of routing in massively dense networks which had been developed within the community of road traffic engineers. Indeed, this approach had already been introduced in 1952 by Wardrop [31] and by Beckmann [4] and is still an active research area among that community, see [6,7,14,17,33] and references therein.

1.3. Our contribution and the paper's structure

We combine in this paper various approaches from the area of road traffic engineering as well as from optimal control theory in order to formulate models for routing in massively dense networks. We further propose a simple novel approach to that problem using a classical device of 2D, singular optimal control [21], based on Green's formula to obtain a simple characterization of least cost paths of individual packets. We end the paper by a numerical example for computing an equilibrium.

The paper starts with a background on the research on massively dense ad hoc networks. In doing so, it is not limited to a specific structure of the cost. However, when introducing our approach based on road traffic tools, we choose to restrict ourselves to static networks (say sensor networks) having a special cost structure characterized by communications through horizontally and vertically oriented directional antennas. The use of directional antennas, by pointing information in a specific direction, allows one to save energy which may result in a longer life time of the network. The nodes are assumed to be placed deterministically. For an application of our approach to omni-directional antennas, see [1]. We solve various types of optimization problems: We consider (i) the global

[1] We note that this approach is restricted to costs that do not depend on the congestion.