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Minimum cost multiple multicast network coding with quantized rates

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ABSTRACT

In this paper, we consider multiple multicast sessions with intra-session network coding where rates over all links are integer multiples of a basic rate. Although having quantized rates over communication links is quite common, conventional minimum cost network coding problem cannot generally result in quantized solutions. In this research, the problem of finding minimum cost transmission for multiple multicast sessions with network coding is addressed. It is assumed that the rate of coded packet injection at every link of each session takes quantized values. First, this problem is formulated as a mixed integer linear programming problem, and then it is proved that this problem is strongly NP-hard on general graphs. In order to obtain an exact solution for the problem, an effective and efficient scheme based on Benders decomposition is developed. Using this scheme the problem is decomposed into a master integer programming problem and several linear programming sub-problems. The efficiency of the proposed scheme is subsequently evaluated by numerical results on random networks.

1. Introduction

In communication networks, routing has long been an important technique for optimizing data transmissions from sources to destinations. In conventional optimal routing problems, each node is capable of forwarding the incoming packets without making any changes in them. Network coding generalizes traditional routing paradigm by allowing nodes to perform arbitrary operations on packets received and generate new output packets. The idea of network coding has its origin in the work of Ahlswede et al. [1], where it was shown that network coding is capable of achieving the maximum-flow minimum-cut bound on the multicast capacity. Li et al. [2] showed that linear network codes are sufficient for the multicast capacity and Jaggi et al. [3] presented a polynomial-time algorithm for constructing such linear codes. The problem of finding a minimum cost multicast scheme in networks using a coded packet approach was addressed by Lun et al. [4]. It was shown that the solution of this problem can be decomposed into two parts: (i) finding the minimum cost subgraph and (ii) determining a code to use over the optimal subgraph. A distributed solution for the second part was provided in [5]. To solve the first part, Lun et al. [4,6] proposed a linear optimization formulation and presented a distributed algorithm using the dual subgradient method to obtain an optimal subgraph. Ghasvari et al. [7] considered the problem of finding a minimum cost multicast subgraph based on network coding, where delay values associated with each link, limited buffer-size of intermediate nodes and link capacity variations over time were taken into account. They proposed a decentralised algorithm using an auxiliary time-expanded network. Wu et al. [8] worked on utility maximization problems, where they developed a primal-subgradient type distributed algorithm based on finding the critical cut. In a similar fashion, there are other algorithms with the aim of finding

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flow subgraphs that guarantee a minimum rate and maximize a utility function [9,10]. However, the algorithms proposed so far in the literature cannot derive quantized (multiples of a basic rate μ) rates over communication links. Koetter et al. [11] pointed out that fractional rates can be well approximated by choosing the time units large enough. But, such capacity scaling will increase the encoding and decoding complexity and introduce large delays at receivers. The delay at receivers will limit the application of network coding in delay sensitive or computational resources limited network applications. Thus, it is necessary that the rates on links to be constrained in order to take quantized values. Cui et al. [12] presented a method for finding a minimum cost subgraph in a single multicast connection with network coding on directed graphs, where packet transmission rates on each link are constrained to be integral. In fact, link rates are constrained to integer multiples of the basic rate, $\mu = 1$. They proposed a greedy algorithm and an LP rounding algorithm, which have approximation ratios k and $2k$, respectively, where k is the number of receivers. In general communication networks, it is not necessary that link rates be integer values, but they generally can only accept quantized values. Moreover, in many applications, multiple multicast sessions governed by their own network constraints share the same network. As the flows do share common links, the single multicast session problems are not independent. In order to find an optimal flow, it is therefore needed to solve the problems in conjunction with each other. Hence, the problem of specifying whether or not a set of multicast connections is feasible becomes more complicated than an equivalent problem having only a single multicast connection. As a result, more effective techniques should be devised to examine the feasibility of a multicast connections set.

In this paper, we consider multiple multicast sessions with intra-session network coding and assume that, instead of one source process at a single node, s, there are M single-rate multicast source processes at nodes $s_1, s_2, \ldots, s_M$, where all receivers in a session, m, receive services at the same rate, R_m. Furthermore, we assume that link rates are constrained to be integer multiples of a positive basic rate μ. Here, the aim is to find minimum cost solutions for the case of general connections, where the rate of the coded packet transmission on each link of every multicast connection is constrained to quantized values. To do so, we first formulate this problem as a mixed integer linear programming (MILP) problem and show that this problem is strongly NP-hard on general graphs, even on series–parallel graphs. Then, we develop a solution based on the Benders decomposition for this problem and implement the proposed approach. The Benders decomposition is an efficient method for solving large-scale MILP problems. Instead of considering all decision variables and constraints of a large-scale problem simultaneously, this approach partitions the original problem into a master integer programming problem and several linear programming sub-problems [13,14]. Since the computational difficulty of MILP optimization problems significantly increases as the number of variables and constraints rises, solving smaller problems with less variables and constraints iteratively can be more efficient than solving a single large problem. Each iteration of the Benders decomposition algorithm acquires a lower bound for the objective value of the original problem. Hence we describe a heuristic procedure to obtain a tight bound in a reasonable computational time. The ability of the proposed approach in reaching the optimal solution is shown through simulations on random graphs.

The remaining of this paper is structured as follows: Section 2 presents a mixed-integer formulation of the proposed problem and its complexity. In Section 3, the proposed problem is solved using the Benders decomposition algorithm. Section 4 introduces valid inequalities in order to reduce the number of iterations in the Benders decomposition algorithm. In Section 5, a heuristic method is used to obtain an upper bound on the optimal value of the objective function. Sections 6 and 7 are allocated to the computational results and conclusion, respectively.

2. Problem definition complexity

2.1. Network model and notations

A communication network consists of a set of directed links connecting transmitters, switches and receivers. It can be represented by a directed graph $G = (V, A)$, where V is a set of nodes and A is a set of links. We denote a link either by a single index, e, or by the directed pair (i, j) of nodes, which represents a lossless point-to-point link from node i to node j. For link $e = (i, j)$, we write $head(e) = j$, and $tail(e) = i$. Each link, e, is associated with two parameters: a non-negative cost, a_e, that denotes the cost per the unit rate of sending coded packets over link $e \in A$, and a non-negative integer capacity, u_e, that denotes the maximum number of packets that can be sent over link e in one time unit.

For node $i \in V$, two link adjacency lists δ_i^+ and δ_i^- are defined as the set of links leaving node i ($tail(e) = i$) and entering node i ($head(e) = i$), respectively.

We now consider a set of multicast sessions, M, in network G. Subsequently, instead of one source process at a single node, s, we consider that there are $|M|$ source processes at nodes s_m, $m \in M$. Each session $m \in M$ is identified by the source–destination pair (s_m, T_m), where s_m is the source node and T_m is the set of receivers of session m. We consider a single-rate multicast, where all receivers in session m receive service at the same rate, R_m, from s_m. Moreover, it is assumed that the data of different sessions are coded independently. Thus, network coding is applied to individual sessions.

We assume all link rates are constrained to be multiples of a positive basic rate μ. The flow rate toward the receiver, r, which belongs to session m and is passing through link e is indicated by $\mu x_e^{m,r}$. As mentioned earlier, network coding generalizes the traditional routing paradigm. In fact, by applying network coding, multicast connections can be established with a significantly lower bandwidth requirement than that of traditional Steiner-tree-based multicast connections [1,6]. Similar to the case of Steiner tree multicast, the consumed bandwidth of each link, e, for the multicast connection, m, is the $\max_{r \in T_m}(\mu x_e^{m,r})$ [6,15]. In a