Computer Networks 55 (2011) 3364–3375

Contents lists available at ScienceDirect

Computer Networks

journal homepage: www.elsevier.com/locate/comnet

Estimation of missing RTTs in computer networks: Matrix completion vs compressed sensing

Ziqian Dong [a,*], Santhanakrishnan Anand [b], Rajarathnam Chandramouli [b]

[a] Department of Electrical and Computer Engineering, New York Institute of Technology, New York, NY 10023, USA
[b] Department of Electrical and Computer Engineering, Stevens Institute of Technology, Hoboken, NJ 07030, USA

ARTICLE INFO

Article history:
Received 8 July 2010
Received in revised form 27 March 2011
Accepted 3 July 2011
Available online 12 July 2011

Keywords:
Estimation
RTT
Compressed sensing
DN matrix completion

ABSTRACT

We estimate the missing round trip time (RTT) measurements in computer networks using doubly non-negative (DN) matrix completion and compressed sensing. The major contributions of this paper are the following: (i) an iterative DN matrix completion that minimizes the mean square estimation error; (ii) mathematical conditions for the convergence of the algorithm; (iii) systematic and detailed experimental comparison of DN matrix completion and compressed sensing for estimating missing RTT estimation in computer networks. To our knowledge, this is the first work that compares the pros and cons of compressed sensing and DN matrix completion for RTT estimation using actual Internet measurement data. Results indicate that compressed sensing provides better estimation in networks with sporadic missing values while DN completion of matrices is more suitable for estimation in networks which miss blocks of measurements. Our proposed DN matrix completion method is one of the first approaches to matrix completion, that minimizes the estimation error.

1. Introduction

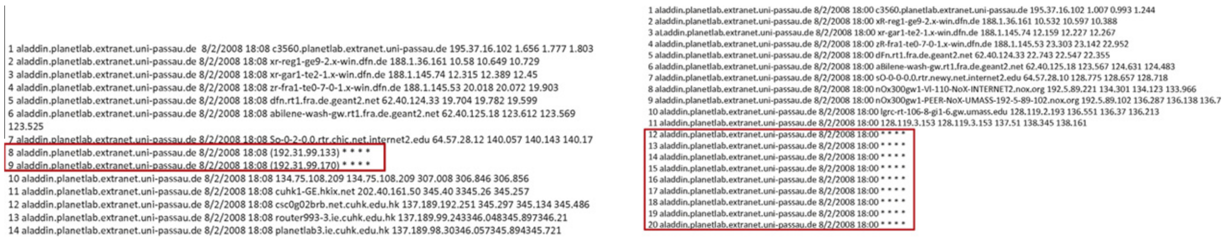
The performance of communication networks largely depends on the instantaneous or the long-term statistical measurements of certain network parameters, e.g., packet-loss [1], end-to-end delay [2], etc. The measured network parameters are used to enable efficient network management. In computer networks, an important network parameter is the end-to-end delay (often measured in terms of round trip time (RTT)) between different nodes in the network. The RTTs, in turn, are used to support different network engineering tasks such as traffic engineering, capacity planning, and support for quality-of-service (QoS).

In many practical scenarios, it may not be possible to obtain the measurements related to some pairs of nodes. For instance, in computer networks, the RTT is measured using *Traceroute* [3] and *ping* [4], by sending probing packets from a source node to a destination node. Internet control message protocol (ICMP) packets are used to measure the RTT between the source and the destination nodes. In practice, many routers and servers block ICMP packets for security reasons. Alternatively, congestions in the routes between nodes could result in packet loss. These make network measurements inaccessible to a large number of intermediate nodes. Fig. 1 shows examples of incomplete RTT measurements when using *traceroute* between Planet-lab [5] nodes `aladdin.planetlab.extranet.uni-passau.de` and `planetlab3.ie.cuhk.edu.hk`. The information enclosed in the rectangle in Fig. 1a and b corresponds to the missing measurements due to *traceroute* being blocked as a security measure and due to route congestion between the nodes, respectively.

Estimation of end-to-end delay between a single source destination pair using past samples was studied in [2]. End-to-end delay prediction using a multiple-model approach was introduced in [1]. Data recovery for patterned data set has been studied in signal processing [6]. Compressed sensing (also known as compressive sampling), is a new theory developed in recent years. Here, signals can

* Corresponding author.
E-mail addresses: ziqian.dong@nyit.edu (Z. Dong), asanthan@stevens.edu (S. Anand), mouli@stevens.edu (R. Chandramouli).

1389-1286/$ - see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.comnet.2011.07.003



(a) Due to security settings at intermediate node

(b) Due to route congestion

Fig. 1. Results of the *traceroute* between Planet-Lab nodes `aladdin.planetlab.extranet.uni-passau.de` and `planetlab3.ie.cuhk.edu.hk`. Missing values correspond to the information enclosed by the rectangle.

be sampled at a rate, significantly less than the Nyquist rate (given by the Nyquist–Shannon Theorem [7,8]), and yet be faithfully recovered [9–13]. Compressed sensing applies ortho-normal sparse representation (using linear transforms like Karhunen–Loeve transform (KLT), discrete Fourier transform (DFT), discrete cosine transform (DCT), etc. [14]), and incoherence measurements of signals, to obtain maximum information using minimum amount of measurements. Detailed information on compressed sensing can be found in [15,16]. Spatio-temporal compressive sensing framework was proposed to estimate the Internet traffic matrix [17], where the authors apply sparsity regularized matrix factorization to exploit the low-rank nature of traffic matrices and their spatio-temporal properties.

For a network with N nodes, the measurement corresponding to each pair of nodes can be represented as an element in an $N \times N$ matrix. The missing information between some pairs of nodes leads to a partially complete measurement matrix. As an example, consider the network shown in Fig. 2. The RTT measurements are available between all pairs of nodes in the sets, $\mathcal{S}_1 = \{A, B, C, D\}$ and $\mathcal{S}_2 = \{D, E, F\}$. This is represented as a solid line in Fig. 2. The RTT measurements between the pairs in the set $\mathcal{S}_3 = \{(A,E),(A,F),(B,E),(B,F),(C,E),(C,F)\}$ are missing (represented as the dashed lines in Fig. 2). Let r_{ij} denote the RTT between nodes i and j in a network. The RTT matrix corresponding to the network shown in Fig. 2, $\mathbf{R}$, can then be written as

$$\mathbf{R} = \begin{bmatrix} 0 & r_{AB} & r_{AC} & r_{AD} & ? & ? \\ r_{BA} & 0 & r_{BC} & r_{BD} & ? & ? \\ r_{CA} & r_{CB} & 0 & r_{CD} & ? & ? \\ r_{DA} & r_{DB} & r_{DC} & 0 & r_{DE} & r_{DF} \\ ? & ? & ? & r_{ED} & 0 & r_{EF} \\ ? & ? & ? & r_{FD} & r_{FE} & 0 \end{bmatrix}, \qquad (1)$$

where ? represents a missing RTT measurement. Recovering the missing information is essentially the completion of the matrix, from the available measurements.

Completion of matrices with specific properties is another area actively researched (e.g., [18–21]). Here, a partially complete matrix with certain property, $\mathcal{P}$, is considered. Then techniques are developed to complete the matrix while retaining the property, $\mathcal{P}$. The property, $\mathcal{P}$, could be positive definiteness, negative definiteness, double non-negativity, double negativity, M-matrix property, etc. [22]. Completion of symmetric inverse M-matrices was discussed in [18]. Doubly non-negative (DN) matrix completion was studied in [19,20], in which the authors discussed the conditions under which a partial DN matrix can have a DN completion. The list of properties for which the matrix completion problem has been solved can be found in [21]. Although the current literature discusses mechanisms for DN matrix completion (e.g., [20]), the provided solutions are not unique and are not optimized to minimize the estimation error. The available solutions could then result in large estimation errors, when directly applied to estimating missing measurements in communication networks. Therefore it is of interest to explore DN matrix completion techniques that not only provide an estimate of the missing measurements, but also minimizes the estimation error.

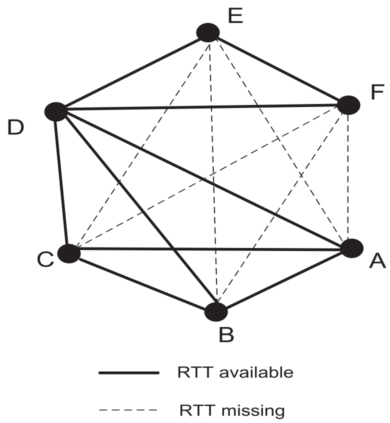


Fig. 2. A network with 6 nodes, with missing RTT measurements between some pairs of nodes.

The major contributions of this paper are the following: (i) an iterative DN matrix completion that minimizes the mean square estimation error; (ii) mathematical conditions for the convergence of the algorithm; (iii) systematic and detailed experimental comparison of DN matrix completion and compressed sensing for estimating missing RTT estimation in computer networks. To our knowledge, this is the first work that compares the pros and cons of compressed sensing and DN matrix completion for RTT estimation using actual Internet measurement data. The proposed method minimizes the Frobenius norm [22] of the estimation error. We describe and prove the sufficient conditions