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On the behavior of stable subnetworks in non-ergodic networks with unreliable nodes

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ABSTRACT

We consider Jackson networks with unreliable nodes, which randomly break down and are under repair for a random time. The network is described by a Markov process which encompasses the availability status and queue lengths vector. Ergodicity conditions for many related networks are available in the literature and can often be expressed as rate conditions. For (reliable) nodes in Jackson networks the overall arrival rate has to be strictly less than its service rate. If for some nodes this condition is violated, the network process is not ergodic. Nevertheless, it is known that in such a situation, especially in large networks, parts of the network (where the rate condition is fulfilled) in the long run stabilize. For standard Jackson networks without breakdown of nodes, the asymptotics of such stable subnetworks were derived by Goodman and Massey [J.B. Goodman, W.A. Massey, The non-ergodic Jackson network, Journal of Applied Probability 21 (1984) 860–869].

In this paper, we obtain the asymptotics of Jackson networks with unreliable nodes and show that the state distribution of the stable subnetworks converges to a Jackson-type product form distribution. In such networks with breakdown and repair of nodes, in general, the ergodicity condition is more involved.

Because no stationary distribution for the network exists, steady-state availability and performance evaluation is not possible. We show that instead assessment of the quality of service in the long run for the stabilizing subnetwork can be done by using limiting distributions. Additionally, we prove that time averages of cumulative rewards can be approximated by state-space averages.

1. Introduction

Our investigation in this note is motivated by the following observation: In large and complex networks, unstable subnetworks often occur due to local overload or due to non-availability of nodes or links, but nevertheless, there co-exist regions where other subnetworks stabilize locally.

The most recent applications, where nodes or links may be not available for some time, are mobile and ad-hoc networks or sensor networks. The emergence of wireless ad-hoc networks, which are built of varying sets of mobile users with wireless communication capabilities without relying on a pre-existing infrastructure, introduces problems concerning availability of transmission nodes in the vicinity of a user. On the other hand, in such networks, overload emerges from too many users entering a region and applying for transmission. So in these networks, the required high quality of service for speech and data transmission has to be guaranteed on the basis of not necessarily reliable network nodes [1].

There are many problems that require an integrated model to study the interplay of performance and availability, e.g., with network control and routing protocols in case of link failures in IP networks [2,3] or with the handling of catastrophic events in mobile cellular networks [4]. A

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related field of research is performance and reliability analysis of sensor networks.

A different area where queueing network models are successfully used to describe the behavior of complex networks is production control in flexible manufacturing systems (FMS) [5]. The investigation of unreliable machines in FMS and the evaluation of system availability has found increasing interest recently.

Our aim is to contribute to performability modeling and evaluation in the framework of generalized Jackson networks, with an emphasis on situations where the global network is no longer stable in the classical sense. We study the interaction of reliability and performance in these networks and contribute to a better understanding of the behavior of networks that cannot approach a classical equilibrium state.

Classical product form network theory is concerned with stations which have completely reliable servers. Usually, when unreliable stations were to be considered, the method of adjusted service rates was used: (1) The reliability of the nodes was computed using pure reliability theoretical methods and yielded the percentage of time the nodes are down. (2) The service speeds of the nodes were then decreased by this factor and the performance evaluation methods for product form networks could be applied, see [6]. The drawback of this method is that the interaction of performance and reliability cannot be studied in a unified model.

Nevertheless, there is a strong need for such studies, and the predominant methods developed to study the interaction of reliability and performance of complex systems in integrated models are simulation and numerical evaluation. An example for this approach is the MOSEL project [7]. MOSEL, a *powerful language for the performance and reliability modeling of computer, communication and manufacturing systems*, was developed under supervision of Gunter Bolch and with many contributions by himself.

Many different models and methods for integrated investigation of reliability and performance of systems are collected under the heading of performability in the survey [8]. Our approach is different from these methods:

We start from results on Jackson networks with unreliable nodes which still have product form equilibrium [9]. But we do not assume that the global network is stable, which may be due to the breakdowns or due to overload of nodes. We describe the network's evolution over time by a Markov process, the states of which encompass the necessary information about the nodes' availability (up or down) and the queue lengths at the nodes (availability–queue-length vector). If the network is not globally stable, this Markov process is not ergodic.

Such networks were investigated under the constraint that all servers are reliable (i.e., no breakdown occurs) by Goodman and Massey [10]. They identified the regions of instability and proved that in these regions the queue-length distributions degenerate to one-point distributions concentrated at ∞, while in the regions of stability the marginal queue lengths converge to a product form distribution.

We continue this investigation in the framework of networks with unreliable stations. Our main result is similar to Goodman and Massey's:

(1) Their method applies in our framework as well and allows to determine the regions of stability and instability.
(2) In the regions of instability the queue-length distributions degenerate to one-point distributions concentrated at ∞.
(3) In stability regions, the marginal distributions of the Markovian state process converge to a product form distribution. The limiting product form distribution is (i) a product with respect to the availability–queue-length vector, and (ii) the queue-length vector exhibits internal product form over the nodes as classical Jackson networks do.

Although we have obtained the limiting availability–queue-length distribution, no stationary distribution for the network exists. Therefore, classical steady-state availability and performance evaluation, even for the subnetwork that stabilizes over time, is not possible.

The results obtained suggest the following substitute procedure: We show that assessment of the quality of service inside the stabilizing part of the network in the long run can be done by using the obtained limiting distributions.

We discuss this in detail for the limiting throughput, but generalization to other performance measures is obvious.

Additionally, we prove that time averages of cumulative rewards associated to the queue-length process of the stabilizing subnetwork can be approximated by state-space averages. In ergodic Markovian networks, this is an immediate consequence of the ergodic theorem for Markov processes. In the framework of our non-ergodic (not Markovian) process, we have to prove this from scratch. Fortunately enough, this can be derived directly from the proof of the main theorem.

The results presented in this paper provide performance indices in explicit form for non-ergodic networks which look exactly like classical product form steady-state distributions. These product form steady states and the algorithms derived from them have been proven to be an indispensable tool set for performance evaluation. Two procedures relying on product form calculus are standard:

- if the necessary modeling assumptions for product form models can be validated, the performance indices are directly obtained,
- if the necessary assumptions for product form models are violated, the algorithms are used to obtain quick answers about main performance characteristics like throughput or system times via approximation in a decomposition-aggregation procedure.

More details are given in [11, Chapters 7 and 8] for the case of exact models and in [11, Chapter 10] for the case of approximating non-product form models by (adjusted) product form algorithms.

Nevertheless, product form models are not a panacea and there are many situations and models where additionally (or instead) simulations or straight forward numerical algorithms have to be used. A common practice often is: First to obtain some raw performance indices from