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Selfishness, collusion and power of local search for the ADMs minimization problem ☆

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ABSTRACT

We consider non-cooperative games in all-optical networks where users share the cost of the used ADM switches for realizing given communication patterns. We show that the two fundamental cost sharing methods, Shapley and Egalitarian, induce polynomial converging games with price of anarchy at most $\frac{5}{3}$, regardless of the network topology. Such a bound is tight even for rings. Then, we show that if collusion of at most k players is allowed, the Egalitarian method yields polynomially converging games with price of collusion between $\frac{3}{2}$ and $\frac{3}{2}+\frac{1}{k}$. This result is very interesting and quite surprising, as the best known approximation ratio, that is $\frac{3}{2}+\epsilon$, can be achieved in polynomial time by uncoordinated evolutions of collusion games with coalitions of increasing size. Finally, the Shapley method does not induce well defined collusion games, but can be exploited in the definition of local search algorithms with local optima arbitrarily close to optimal solutions. This would potentially generate PTAS, but unfortunately the arising algorithm might not converge. The determination of new cost sharing methods or local search algorithms reaching a compromise between Shapley and Egalitarian is thus outlined as being a promising and worth pursuing investigating direction.

1. Introduction

1.1. Background

All-optical networks have been largely investigated in recent years due to the promise of data transmission rates several orders of magnitudes higher than current networks [6,7,22,24]. Major applications are in video conferencing, scientific visualization and real-time medical imaging, high-speed supercomputing and distributed computing [11,22].

The key to high speeds in all-optical networks is to maintain the signal in optical form, thereby avoiding the prohibitive overhead of conversion to and from the electrical form at the intermediate nodes. The high bandwidth of the optical fiber is utilized through *wavelength-division multiplexing*: two signals connecting different source-destination pairs may share a link, provided they are transmitted on carriers having different wavelengths (or colors) of light. The optical spectrum being a scarce resource, given communication patterns in different topologies are often designed so as to minimize the total number of used colors, also as a comparison with the trivial lower bound provided by maximum load, that is the maximum number of connection paths sharing a same physical edge [3,19].

When the various parameters comprising the switching mechanism in these networks became clearer, the focus of

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studies shifted, and today a large portion of research concentrates with the total hardware cost. This is modelled by considering the basic electronic switching units of the electronic add-drop multiplexer (ADM) and focusing on the total number of these hardware components. Each lightpath uses two ADMs, one at each endpoint. If two non-overlapping lightpaths are assigned the same wavelength and are incident to the same node, then they can use the same ADM. Thus, an ADM may be shared by at most two lightpaths. The problem of minimizing the number of ADMs was introduced in [21] for ring networks. For such a topology it was shown to be NP-complete [13] and an approximation algorithm with approximation ratio 3/2 was presented in [9] and improved in [31,14] to $10/7+\epsilon$ and 10/7 respectively. For general topologies an algorithm with approximation ratio 8/5 is presented in [13]. For the same problem, algorithms with approximation ratio $3/2+\epsilon$ were provided in [8,17].

In a distributed and decentralized environment characterizing an optical communication network, besides the classical design of centralized algorithms optimizing the resources utilization, the analysis of the uncooperative interaction between the network users and the design of distributed algorithms call for more research effort. On this respect, Game Theory and the associated concept of Nash equilibria [28] have recently emerged as a powerful tool for modelling and analyzing such a lack of coordination. In this setting, each communication request is handled by an agent (or player) selfishly performing *moves*, i.e. changing her routing strategy in order to maximize her own benefit. A Nash equilibrium is a solution of the game in which no agent gains by unilaterally changing her routing strategy. If Nash equilibria are reached in a polynomial number of selfish moves, and finding an improving user move is a problem solvable in polynomial time, such an uncooperative process naturally defines a distributed polynomial time algorithm. However, due to the lack of cooperation among the players, Nash equilibria are known not to always optimize the overall performance. Such a loss [10,1] has been formalized by the so-called *price of anarchy* (resp. *optimistic price of anarchy*), defined as the ratio between the cost of the worst (resp. best) Nash equilibrium and the one of an optimal centralized solution.

There exists a vast literature on Nash Equilibria in communication networks [25,29]. The problem of investigating the existence and performance of Nash equilibria in all-optical networks has been first considered [4,5] with respect to the minimization of the number of used wavelengths. In such a setting, a service provider has to satisfy a given set of point-to-point communication requests, charging each of them a cost depending on its wavelength and on the wavelengths of the other requests met along its path in the network. Each request is issued by a non-cooperative agent interested only in the minimization of her own cost. In a similar setting [20], the authors focus on the complexity of recognizing and computing Nash equilibria.

Even if in non-cooperative games players are usually considered to act selfishly and independently, an interesting investigated issue is the one of collusion. Roughly speaking, collusion allows two or more players forming a coalition to come to an agreement in order to obtain a gain by changing at the same time their strategies. In this framework, a Nash equilibrium is a solution in which there exists no coalition of players having convenience in changing their strategies. The lack of performance with respect to the optimal solution has been measured by the *price of collusion* [18,23], where the authors focused on a particular class of games, the congestion games, assuming the players partitioned into sets of coalitions. Earlier [2,26], the authors provided other related equilibrium concepts, such as strong or coalition-proof equilibria, ensuring that coalitions have no incentive to form. Coalitions have also been considered from the perspective of the algorithmic mechanism design, with emphases on group-strategyproof mechanisms [27].

1.2. Our contribution

Following the research direction outlined in [16], in this paper we are interested in analyzing the non-cooperative scenario in which the users of an optical network interact sharing the cost of the used hardware components. More precisely, we focus on ADM switches, considering the game in which their total cost is divided between the users according to two fundamental cost sharing methods: the Shapley [32] method, in which the agents using an ADM pay for it by equally splitting its cost, and the Egalitarian one, where the whole hardware cost is equally split among all the players.

We show that without collusion the two cost sharing methods are equivalent and induce games always convergent in polynomial time, i.e. the players always reach an equilibrium configuration within a polynomial number of selfish moves. Moreover, we prove that the arising price of anarchy is at most $\frac{5}{3}$ regardless of the network topology, and that such a result is tight even for rings. This result is very interesting, as it matches the performance of three different algorithms [13,8].

Under the assumption that the collusion of at most k players is allowed, only the Egalitarian cost sharing method yields a well-founded definition of induced game. We show that such a game is still convergent, and its price of collusion is $\frac{3}{2}+\frac{1}{k}$. This result is quite surprising, as the best known approximation ratio reached by a centralized algorithm [8], that is $\frac{3}{2}+\epsilon$, can be achieved in polynomial time by uncoordinated evolutions of collusion games with coalitions of increasing size. As already remarked, this has the additional appreciable effect of yielding a polynomial time approximation algorithm of distributed nature with the best so far achievable optimization performance.

Finally, always under the assumption of collusion, the Shapley method does not induce well defined games, but it can be exploited in the definition of proper neighborhoods in local search algorithms. The arising local optima are arbitrarily close to optimal solutions, that is at most $1+\frac{2}{k}$ times the optimum, thus potentially generating distributed PTAS; unfortunately, the arising algorithms might not converge and such local optima might even not exist at all. However, this sheds some light on the effectiveness of local search in improving the current approximation fac-