Advances in Water Resources 98 (2016) 47–69

Contents lists available at ScienceDirect

Advances in Water Resources

journal homepage: www.elsevier.com/locate/advwatres

A non-hydrostatic pressure distribution solver for the nonlinear shallow water equations over irregular topography

Costanza Aricò*, Carlo Lo Re

Dipartimento di Ingegneria Civile, Ambientale, Aerospaziale, dei Materiali, Università di Palermo, Viale delle Scienze, 90128, Palermo, Italy

ARTICLE INFO

Article history:
Received 13 April 2016
Revised 20 October 2016
Accepted 21 October 2016
Available online 22 October 2016

Keywords:
Dynamic pressure
Shallow waters
Dispersive process
Finite volume
Wetting and drying
Unstructured grid

ABSTRACT

We extend a recently proposed 2D depth-integrated Finite Volume solver for the nonlinear shallow water equations with non-hydrostatic pressure distribution. The proposed model is aimed at simulating both nonlinear and dispersive shallow water processes. We split the total pressure into its hydrostatic and dynamic components and solve a hydrostatic problem and a non-hydrostatic problem sequentially, in the framework of a fractional time step procedure. The dispersive properties are achieved by incorporating the non-hydrostatic pressure component in the governing equations. The governing equations are the depth-integrated continuity equation and the depth-integrated momentum equations along the *x*, *y* and *z* directions. Unlike the previous non-hydrostatic shallow water solver, in the *z* momentum equation, we retain both the vertical local and convective acceleration terms. In the former solver, we keep only the local vertical acceleration term. In this paper, we investigate the effects of these convective terms and the possible improvements of the computed solution when these terms are not neglected in the governing equations, especially in strongly nonlinear processes. The presence of the convective terms in the vertical momentum equation leads to a numerical solution procedure, which is quite different from the one of the previous solver, in both the hydrostatic and dynamic steps. We discretize the spatial domain using unstructured triangular meshes satisfying the Generalized Delaunay property. The numerical solver is shock capturing and easily addresses wetting/drying problems, without any additional equation to solve at wet/dry interfaces. We present several numerical applications for challenging flooding processes encountered in practical aspects over irregular topography, including a new set of experiments carried out at the Hydraulics Laboratory of the University of Palermo.

© 2016 Elsevier Ltd. All rights reserved.

1. Introduction

In recent decades, the NonLinear Shallow Water Equations (NLSWEs) with hydrostatic pressure distribution have been widely used to simulate wave processes in inland shallow waters (e.g., rivers and estuaries) or in water wave transformations in nearshore zones (from the surf zone to the shoreline) for coastal processes. The primary reasons for their use are their simplicity and accuracy over irregular topography. Unfortunately, the NLSWEs with hydrostatic pressure distribution are unable to simulate some dispersive features of water waves (e.g., waves with different frequencies travel at different speeds) (Walters, 2005; Wei and Jia, 2013; Yamazaki et al., 2008, 2011) or secondary free-surface undulations at wave fronts or tails (undular bores and shocks generated by dam-break flows or tsunamis), which are dispersive in nature (Soares-Frazao and Zech, 2002; Kim and Lynett, 2011).

Generally, a good numerical model for water waves should guarantee a balance between the frequency dispersion and nonlinearity.

In recent decades, several methods for solving the 3D RANS equations have been proposed, e.g., RANS models (OpenFOAM® Foundation 2013), Smoothed Particle Hydrodynamics methods (Dalrymple and Rogers, 2006) and Volume Of Fluids methods (Hirt and Nichols, 1981), but these approaches generally have very high computational costs.

Among the depth-integrated equations models, the Boussinesq-type models (BTMs) and NLSWEs with a non-hydrostatic pressure distribution are two candidates that guarantee a good compromise between nonlinearity and frequency dispersion.

Weak nonlinearity and dispersion affect the classical formulation of the BTMs (Peregrine, 1967), and the high-order BTMs proposed to overcome these problems present complex numerical discretization and high computational burdens (Brocchini, 2013; Yamazaki et al., 2008). In general, due to the high-order partial derivative terms, the BTMs suffer from the use of extra terms and empirical criteria for wave breaking simulation and energy dissipa-

* Corresponding author.
E-mail address: costanza.arico@unipa.it (C. Aricò).

http://dx.doi.org/10.1016/j.advwatres.2016.10.015
0309-1708/© 2016 Elsevier Ltd. All rights reserved.

tion, as well as the complex wetting/drying procedures (Brocchini, 2013; Wei and Jia, 2013).

The use of NLSWEs models with non-hydrostatic pressure distribution is a relatively new approach. These models account for the vertical acceleration using the non-hydrostatic pressure, and they include the non-hydrostatic effects by splitting the pressure term into its hydrostatic and dynamic (or non-hydrostatic) components (Casulli and Stelling, 1998). Two fractional-step procedures are generally applied: the pressure projection and pressure correction methods (Cui et al., 2002 and cited references). From now on, “hydrostatic” and “non-hydrostatic” models refer to the models with hydrostatic and non-hydrostatic pressure distribution, respectively.

Wave breaking simulation is a challenging topic for depth-integrated NLSWEs models. It is well known that if the governing equations are written in a conservative form, the hydrostatic NLSWEs models properly simulate discontinuous flows (e.g., shocks, hydraulic jumps, and bores) (LeVeque, 1992; Toro, 2009; Stelling and Duinmeijer, 2003). Many shock-capturing Godunov-type Finite Volume (FV) solvers have been proposed to solve the hydrostatic NLSWEs during the last three decades (Alcrudo and Garcıa-Navarro, 1993; Toro, 2009; LeVeque, 1992). The fractional step methodology provided by the pressure projection/correction methods is a suitable approach for developing non-hydrostatic shock-capturing NLSWEs models, but only a few non-hydrostatic models have the desired shock-capturing capability (Stelling and Zijlema, 2003). Due to the general complexity and high computational effort of the Godunov-type FV methods (Fang et al., 2014; Zijlema and Stelling, 2008), some authors adopt such schemes for the hydrostatic part of the governing equations and use Finite Difference methods to handle the dynamic part (Fang et al., 2014; Ma et al., 2012; Stelling and Zijlema, 2003; Zijlema and Stelling, 2008).

Accurate modelling of wetting/drying (WD) processes is another basic aspect for flooding or wave run-up simulations. WD techniques can be classified into Lagrangian and Eulerian approaches (Funke et al., 2011). In real-case applications, the Eulerian methods, which use a fixed mesh, are generally more attractive than the Lagrangian methods, which involve interface tracking and mesh adaptation to a changing computational domain. The main drawbacks of some of the most common WD techniques (e.g., mass imbalance at wet/dry interface and computational burden) have been addressed in previous studies (Brocchini et al., 2002; Gourgue et al., 2009; Zijlema and Stelling, 2008).

We present a depth-integrated, non-hydrostatic NLSWEs FV model in which the governing equations are written in a conservative form. The dynamic pressure terms and vertical momentum equation account for the dispersion. We solve the governing equations by applying a fractional time step procedure, where a hydrostatic problem and a non-hydrostatic problem are sequentially solved. The dynamic pressure terms in the momentum equations are neglected when solving the hydrostatic problem and are retained in the non-hydrostatic problem, allowing for adjustment of the flow field with respect to the one computed by the hydrostatic step. The proposed model is shock-capturing, the WD treatment is implicitly embedded, and no additional equation has to be solved at the wet/dry interfaces. The model is “well-balanced", indicating it preserves both the “water at rest" condition (Bermudez and Vazquez, 1994) and a general equilibrium condition with moving water (non-zero flow velocity).

The hydrostatic problem is solved by a prediction-correction scheme. We use the MArching in Space and Time (MAST) procedure (Aricò and Tucciarelli, 2007a; Aricò et al., 2007, 2013a) to solve the hydrostatic prediction problem. The computational cells are sequentially solved throughout the domain after their ordering at the beginning of each time iteration. In the corrector step of the hydrostatic problem, as well as in the non-hydrostatic problem, we solve a large linear system for the unknown water levels and dynamic pressures, respectively.

Due to their ability to fit arbitrary geometries and irregular natural boundaries, the spatial domain is discretized with unstructured triangular meshes that satisfy the Delaunay property.

The paper is organized as follows. We provide motivations for the present work in Section 2. The governing equations are presented in Section 3. In Section 4, we outline the general formulation of the proposed numerical procedure. Numerical details of the hydrostatic and non-hydrostatic steps, as well as the boundary conditions, are presented in Section 5. The model properties (e.g., the local and global mass balance, well-balanced property for a general condition of moving water at equilibrium, *C*-property, computational burden, convergence order, …) are briefly presented and discussed at the end of the same section and in the Appendices in the file “Appendices-doc" in the supplementary materials. Finally, in Section 6, we present several model applications. These applications are aimed at highlighting the capability of the model to simulate challenging flooding processes that are widely encountered in practical aspects, e.g., dam-break, solitary wave/tsunami run-up, wetting/drying and wave breaking over irregular topography. We also validate the model with a new set of lab experiments performed at the Hydraulics Laboratory of the Department of Civil Engineering of the University of Palermo.

2. Main motivations of the present work

The present work builds on a previous paper (Aricò et al., 2016) in which the authors proposed a numerical solver for the NLSWEs with a non-hydrostatic pressure distribution. In the vertical momentum equation, both the vertical advection and dissipation terms are neglected so that the non-hydrostatic pressure depends essentially on the vertical (local) acceleration of the water column. Other authors have adopted the same hypothesis (Lu et al., 2015; Walters, 2005; Wei and Jia, 2013; Yamazaki et al., 2008; Yamazaki et al., 2011; Zijlema and Stelling, 2008). In Aricò et al. (2016), a hydrostatic problem and a non-hydrostatic problem are sequentially solved in the framework of a fractional time step procedure (Cui et al., 2002), which is similar to the present work. The authors in Aricò et al. (2016) solve the hydrostatic problem for the unknown variables water level and horizontal specific flow rate components. The vertical momentum equation plays the role of a “closure relationship" between the dynamic pressure and vertical flow rate component in the non-hydrostatic problem, where the horizontal and vertical momentum equations are combined in the local divergence-free continuity equation (Aricò et al., 2016). In the present work, we propose a new mathematical formulation of the vertical momentum equation, where the dynamic pressure depends on both the local acceleration and convective terms. From a physical point of view, this means that the weight of the water column supported by the adjacent water columns is not negligible compared with the vertical acceleration of the water column. With respect to the previous work (Aricò et al., 2016), we have one additional unknown variable in the hydrostatic problem, which is the mean vertical flow rate component, and one additional governing equation, which is the vertical momentum equation. The numerical solution could benefit from these new vertical convective acceleration terms, mainly in strongly nonlinear processes. As shown in the next sections, the vertical and horizontal flow rate components are combined in the local divergence-free continuity equation during the non-hydrostatic step, as in Aricò et al. (2016), as well as, according to the new approach, during the hydrostatic step. In Section 6, we provide several comparisons between the numerical solutions obtained with and without the convective vertical acceleration terms.

Download English Version:

https://daneshyari.com/en/article/4525220

Download Persian Version:

https://daneshyari.com/article/4525220

Daneshyari.com