Advances in Water Resources 91 (2016) 37–45

Contents lists available at ScienceDirect

Advances in Water Resources

journal homepage: www.elsevier.com/locate/advwatres

# Optimal control solutions to sodic soil reclamation

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ARTICLE INFO

*Article history:*
Received 15 January 2016
Accepted 23 February 2016
Available online 11 March 2016

*Keywords:*
Sodicity
Reclamation
Salinity
Optimal control
Irrigation
Rehabilitation
Amelioration

ABSTRACT

We study the reclamation process of a sodic soil by irrigation with water amended with calcium cations. In order to explore the entire range of time-dependent strategies, this task is framed as an optimal control problem, where the amendment rate is the control and the total rehabilitation time is the quantity to be minimized. We use a minimalist model of vertically averaged soil salinity and sodicity, in which the main feedback controlling the dynamics is the nonlinear coupling of soil water and exchange complex, given by the Gapon equation. We show that the optimal solution is a bang–bang control strategy, where the amendment rate is discontinuously switched along the process from a maximum value to zero. The solution enables a reduction in remediation time of about 50%, compared with the continuous use of good-quality irrigation water. Because of its general structure, the bang–bang solution is also shown to work for the reclamation of other soil conditions, such as saline–sodic soils. The novelty in our modeling approach is the capability of searching the entire "strategy space" for optimal time-dependent protocols. The optimal solutions found for the minimalist model can be then fine-tuned by experiments and numerical simulations, applicable to realistic conditions that include spatial variability and heterogeneities.

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## 1. Introduction

In arid and semi-arid areas, where good-quality water for irrigated agriculture is scarce, inadequate management of soil and water resources can lead to soil sodification [1]. Soil sodicity is characterized by a relative high concentration of sodium cations in the exchange complex or in the soil water, causing negative effects on the physical and chemical properties of the soil. Among the physical changes are the breakdown of macroaggregates (slaking), the release of individual clay platelets from aggregates (dispersion), and surface crusting, which have a detrimental impact on hydraulic conductivity, infiltration rate, seedling emergence and water holding capacity [2–4]. Chemical effects of sodicity include specific ion deficiencies and toxicities [5].

Several amelioration methods exist for reducing the relative amount of sodium in the soil, including the use of chemical amendments (most commonly calcium-based) [6,7], application of high electrolyte water [8], and phytoremediation [9]. For a review on the amelioration of sodic soils see [10]. Sodic soil reclamation can be very costly and resource intensive (in terms of water, amendments, time, etc.), although arguably not as expensive as a "no action" policy [11]. Therefore, from an economic and environmental perspective, it is important to find strategies that optimize resource usage.

The main approaches used so far to tackle this challenge are experiments and computer simulations. On the one hand, field and laboratory experiments have been employed to compare different reclamation and management strategies for specific soil conditions, and determine the ones that "most effectively" reduce sodicity and improved soil structure [12–19]. On the other hand, computer simulations of unsaturated water flow and solute transport have the advantage of being able to examine the effectiveness of different reclamation methods for various soil conditions with little costs and in much faster times, compared to experiments [20–22].

However, both experiments and cumbersome computer simulations have limited probing power with regard to time-dependent reclamation strategies. Of the uncountable ways one can choose to reclaim sodic soils, these two approaches can usually only compare the outcomes of changing one or more parameters (called control parameters), while keeping them fixed in time throughout the experiment or simulation.

In order to be able to compare a continuum of scenarios and reclamation protocols, in this paper we make use of optimal control theory. This allows us to find optimal reclamation strategies in the entire "strategy space", where the control parameter can be continuously changed in time. For this, we introduce a minimalist model for the dynamics of soil salinity and sodicity [23], that

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http://dx.doi.org/10.1016/j.advwatres.2016.02.014
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is amenable to analysis and that is suitable for solving the optimal control problem, while retaining the essential elements of the physical problem. In this model, we assume that irrigation is the main water input (e.g., negligible precipitation during a Mediterranean dry season), and that the agricultural soil is relatively homogeneous, so that the variables may be averaged over the spatial dimensions. Our main goal here is to show that optimal control theory can be a very useful tool in the problem of degraded soil reclamation, and the specific model in question provides a convenient test case for our approach.

Optimal control theory has been used in many modern environmental challenges, such as the management of soil and water resources [24], soil erosion [25], pollution [26], forest carbon sequestration [27] and urban drainage systems [28]. In the case of agricultural systems, its use has been mainly focused on irrigation water allocation [29–31] and greenhouse management [32,33]. To the best of our knowledge, however, this is the first study on optimal control of sodic soil reclamation.

Within the broader topic of sodic soil reclamation, this work concerns the problem of finding a rehabilitation protocol that allows the reclamation of a sodic soil in the least possible time, by means of irrigation with calcium-based amendments. Thus, the amelioration process is viewed as an optimal control problem [34]: the amount of calcium added to irrigation water is the control, and the optimal strategy is the time-dependent addition of calcium that takes the system from a sodic condition to a desired "normal" soil target, while minimizing a "cost", in our case, the total time. In the course of this paper we will show that the optimal strategy is able to cut by about half the reclamation time (also the amount of irrigation water used), compared to simply using good-quality water to flush sodium cations from the root zone. Although highly idealized, this theoretical estimate provided by control theory offers an upper bound that can serve as a benchmark in actual rehabilitation efforts in the field [10].

The paper is structured as follows. Section 2 describes the model for the dynamics of soil salinity and sodicity. In Section 3 the model is linearized and, by using the tools of optimal control theory, the optimal strategy for the control is calculated analytically. Appendix A, at the end of the paper, describes the more cumbersome arguments involved in the analysis of Section 3. Finally, in Section 4, we discuss the conclusions and future directions.

## 2. Dynamics of salinity and sodicity in the soil

We will briefly review the differential equations that govern the dynamics of soil salinity and sodicity. A detailed description of the model can be found in [23].

The dynamics of salt and sodium cations in the root zone is modulated by the dynamics of water in the soil. We assume here that irrigation is the dominant input of water. This assumption is reasonable when precipitation is negligible, as in a Mediterranean dry season or in case of a greenhouse, and that there is no upward flux of water from the water table due to capillarity. The irrigation rate, denoted by $I$, is constant in time, and its electrolyte concentration is $C_I$. For analytical tractability, we also consider the evapotranspiration rate $T$ to be constant in time, a reasonable assumption in irrigated soils. Both $I$ and $T$ are given in mm/d or L/m$^2$/d. We model here a flat and relatively homogeneous agricultural plot, therefore all the variables are vertically lumped over the rooting depth $Z_r = 0.4$ m. For moderate irrigation rates, no ponding will occur, and the steady-state percolation reads simply $I - T$. Writing the percolation function as $L(s) = rK_s s^c$ [35], we can find the steady-state soil water volume per unit area $w$ (L/m$^2$) as

$$w = nZ_r\left(\frac{I-T}{rK_s}\right)^{1/c}, \tag{1}$$

where $n$ is the porosity, $K_s$ is the saturated hydraulic conductivity, and the parameter $c$ depends on the soil properties [36]. The parameter $r$ modulates the saturated hydraulic conductivity as the salinity and sodicity of the soil change. We will leave the discussion on this dependence to Section 2.2, after the salinity and sodicity variables are introduced.

By writing the balance equation for salt, represented by the total amount of sodium and calcium cations in the root zone, one obtains an equation for the electrolyte concentration of soil water $C$ [23],

$$\frac{dC}{dt} = \frac{IC_I}{w} - \frac{I-T}{w}C, \tag{2}$$

where both $C$ and $C_I$ are in millimoles of charge per liter[1], or mmol$_c$/L. The first term in the right-hand side of Eq. (2) is the salt input due to irrigation, and the second term is the salt output due to water percolation $I - T$ to deeper soil layers. We consider the rehabilitation efforts to take place before the growing season, therefore the equation above does not include salt uptake by plants.

This linear equation has a characteristic time scale $\tau_C = w/(I - T)$ in which the system converges to its steady state $C^\bullet = IC_I/(I - T)$.

Cations in the soil water can replace readily exchangeable cations adsorbed to soil particles, in a process called cation exchange. This reaction has a time scale of minutes to hours, while salinity and sodicity processes take place at much longer time scales of weeks to months. For this reason, we assume the exchange reaction to be in a local thermodynamic equilibrium, which in turn warrants the use of the well known Gapon equation [23,37], linking the sodium cation in the soil water to that in the exchange complex.

The relative amount of sodium in the exchange complex is simply the ratio of sodium cation concentration to the total concentration of adsorbed cations (or the cation exchange capacity, $C_{\text{CEC}}$). This quantity is called the equivalent fraction of sodium in the exchange complex, denoted here by $E$, or when expressed as a percentage, the exchangeable sodium percentage (ESP). The relative amount of sodium in the irrigation water is the equivalent fraction $E_I$, while the equivalent fraction of sodium in the soil water is denoted here by $E_s$.

Using the equivalent fractions of sodium defined above, the Gapon equation can be written as

$$\frac{E}{1-E} = K_g\sqrt{2C}\frac{E_s}{\sqrt{1-E_s}}, \tag{3}$$

where $K_g$ is the Gapon selectivity coefficient. The value of $K_g$ is soil-specific, ranging from 0.0072 to 0.01740 (mmol$_c$/L)$^{-1/2}$ [38,39]. Here we use $K_g = 0.01475$ (mmol$_c$/L)$^{-1/2}$, which represents a mean behavior of 59 soil samples of varied origin, as reported in 1954 by the United States Salinity Laboratory Staff [40]. In this report, Eq. (3) is presented as $\text{ESR} = K_g\,\text{SAR}$, where $\text{ESR} = E(1-E)^{-1}$ is the exchangeable sodium ratio, and $\text{SAR} = [\text{Na}^+][\text{Ca}^{2+}]^{-1/2}$ is the sodium adsorption ratio, and the brackets denote molar concentration in mmol/L. The sodicity of a solution is usually measured by the SAR, but in this paper we use the equivalent fraction of sodium for modeling convenience. However, the SAR can be easily related to the right-hand side of Eq. (3), by using the expression $\text{SAR} = \sqrt{2C_z}E_z(1-E_z)^{-1/2}$, where $C_z$ and $E_z$ are the electrolyte concentration and exchangeable sodium fraction of the solution $z$ in question (either soil solution or irrigation water, in our case).

[1] Note that the quantity mmol$_c$/L coincides with the SI units mol$_c$/m$^3$.

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