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Specific storage and hydraulic conductivity tomography through the joint inversion of hydraulic heads and self-potential data

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ABSTRACT

Transient hydraulic tomography is used to image the heterogeneous hydraulic conductivity and specific storage fields of shallow aquifers using time series of hydraulic head data. Such ill-posed and non-unique inverse problem can be regularized using some spatial geostatistical characteristic of the two fields. In addition to hydraulic heads changes, the flow of water, during pumping tests, generates an electrical field of electrokinetic nature. These electrical field fluctuations can be passively recorded at the ground surface using a network of non-polarizing electrodes connected to a high impedance ($>$ 10 MOhm) and sensitive (0.1 mV) voltmeter, a method known in geophysics as the self-potential method. We perform a joint inversion of the self-potential and hydraulic head data to image the hydraulic conductivity and specific storage fields. We work on a 3D synthetic confined aquifer and we use the adjoint state method to compute the sensitivities of the hydraulic parameters to the hydraulic head and self-potential data in both steady-state and transient conditions. The inverse problem is solved using the geostatistical quasi-linear algorithm framework of Kitanidis. When the number of piezometers is small, the record of the transient self-potential signals provides useful information to characterize the hydraulic conductivity and specific storage fields. These results show that the self-potential method reveals the heterogeneities of some areas of the aquifer, which could not been captured by the tomography based on the hydraulic heads alone. In our analysis, the improvement on the hydraulic conductivity and specific storage estimations were based on perfect knowledge of electrical resistivity field. This implies that electrical resistivity will need to be jointly inverted with the hydraulic parameters in future studies and the impact of its uncertainty assessed with respect to the final tomograms of the hydraulic parameters.

1. Introduction

Groundwater investigations, management for water supply, and environmental remediation of contaminated aquifers require the knowledge of the 3D distributions of the hydraulic conductivity, K, and the specific storage, S_s. Obtaining the needed high-resolution S_s and K fields is however a tedious task (e.g., [18,60,61]). The hydraulic tomography method was developed to use hydraulic head time series, from pumping tests, to estimate these fields using steady-state and/or transient conditions (e.g., [1,7,8,10,11,13,23,54,63]).

Performing tomography of the K- and S_s-fields can be formulated as an inverse problem. This inverse problem is known to be both ill-posed and generally under-determined. One problem with hydraulic tomography is to constrain sufficiently the inversion problem with enough data taken over the region of interest. To better constrain the inversion process, additional data from different surface geophysical methods can be used (e.g., [6,14,20,21,40]). Soueid Ahmed et al. [51] recently developed a method to image hydraulic conductivity from the joint inversion of hydraulic head and self-potential data for pumping tests in steady-state conditions. The idea underlying this approach can be explained as follows. The flow of the pore water drags the excess of electrical charges contained in the electrical diffusive layer coating the surface of minerals (e.g., [24,25,43]). This drag generates a source current density, called the streaming current density of electrokinetic origin (i.e., due to the relative motion between the charged skeleton and the pore water of the material). In turn, this source current density is a source term in the Maxwell equations generating electrical and magnetic field disturbances that can be remotely recorded. The electrical potential associated with the flow of the ground water is called the streaming potential. The self-potential method is a non-intrusive geophysical method based on the

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remote measurements of this electrical field at the ground surface or in boreholes. Other types of self-potential signals may exist, in addition to the streaming potential, for instance associated with corrosion of the metallic casing of the wells but they can be easily subtracted from the measurements (see [50] for a field case study).

In self-potential measurements, the sensors are non-polarizable electrodes connected to a multichannel voltmeter of high impedance (> 10 MOhm) and high sensitivity (0.1 mV). Due to its high sensitivity to the flow of the ground water and its affordable cost, the self-potential method has been used in several studies to detect fluid flow and to characterize hydraulic properties of porous media (e.g., [17,28,31,32,37–39,44,50,56,57]). Bogoslovsky and Ogivly [3] pioneered the use of the self-potential method for pumping tests. Afterwards, Rizzo et al. [50] developed a semi-quantitative method to interpret self-potential signals during pumping tests. Revil et al. [44] applied this approach to the self-potential data associated with ground water flow in a buried paleo-channel. Malama et al. [33,34] derived semi-analytical solutions for the self-potential data associated with pumping tests both in confined and unconfined aquifers under the assumption of homogeneous aquifers. Recently, Ozaki et al. [38] estimated for the first time 2D hydraulic conductivity and specific storage fields (along vertical cross-sections) from the joint inversion of self-potential and pumping tests data. They demonstrated, on a synthetic case, that the estimated specific storage was more sensitive at the beginning of the pumping test. This is in agreement with several works available in the hydrologic literature (e.g., [59]). Soueid Ahmed et al. [51] investigated the usefulness of adding self-potential to hydraulic head data in order to recover the hydraulic conductivity field in steady-state conditions. When reconstructing the hydraulic conductivity field, the self-potential method can help solving the issue of the lack of hydraulic information because of its distinct sensitivity map with respect to classical head data (i.e., the sensitivity pattern of self-potential to K is very different from the sensitivity of head to K).

In the present study, we extend the work of Soueid Ahmed et al. [51] to the transient regime to estimate the specific storage field in addition to the hydraulic conductivity field. We compared the results obtained from the hydraulic tomography alone and from the combination of the self-potential method and the hydraulic tomography during a series of transient pumping/injection tests. We use a petrophysical formulation of the material properties in the forward modeling of the self-potential field, which is different from the one used by Ozaki et al. [38,39]. More explicitly, we use the relationship developed by Jardani et al. [26] allowing the computation of the self-potential signals from an effective volumetric charge density derived directly from the hydraulic conductivity itself through a linear relationship between the log of these two parameters. Such formulation is convenient because it reduces the number of petrophysical parameters needed to be determined during the inversion process. In addition, this formulation is valid for a broad range of lithologies (see [48], for a recent discussion regarding this point). For karstic systems, we need to account for the effect of the Reynolds number [5]. We also use the adjoint-state method [13,30,55,63] to evaluate the sensitivities of the hydraulic heads and self-potential signals to change in hydraulic conductivity and specific storage. The adjoint method is known to be more accurate than the finite differences method and permits significant gain in runtime efficiency.

2. Forward problem

2.1. Forward hydraulic flow problem

The ground water flow in a saturated, heterogeneous aquifer, under the transient regime is governed by the following diffusion equation

$$S_s \frac{\partial h}{\partial t} - \nabla \cdot (K \nabla h) = w, \tag{1}$$

where h is the head (m), S_s is the specific storage field (m^{-1}), K is the hydraulic conductivity field (ms^{-1}), t is the time (s), and w (s^{-1}) is the source/sink term corresponding to injection/extraction rate (in volume per unit volume per unit time, s^{-1}). Eq. (1) is subject to the following boundary and initial conditions

$$h = h_D \; on \; \Gamma_D, \tag{2}$$

$$\hat{\mathbf{n}} \cdot K \nabla h = 0 \; on \; \Gamma_N, \tag{3}$$

$$h = h_0 \; when \; t = t_0, \tag{4}$$

where h_D is the hydraulic head imposed at the Dirichlet boundary Γ_D, Γ_N is the Neumann boundary, $\hat{\mathbf{n}}$ is a unit vector normal to Γ_N and h_0 is the initial water head. Neumann boundary conditions are assigned to the top and bottom boundaries while Dirichlet boundary conditions are assigned to the remaining boundaries of the simulation domain.

2.2. Forward self-potential problem

In an isotropic heterogeneous media, the total current density $\mathbf{j}$ (in $A\,m^{-2}$) defines the total flux of charge carriers (in C $m^{-2}s^{-1}$). It is the sum of a conductive current (given by Ohm's law) and a source current density of advective nature, the streaming current, $\mathbf{j}_S$ (e.g., [26,42])

$$\mathbf{j} = -\sigma \nabla \varphi + \mathbf{j}_S. \tag{5}$$

The streaming current is due to the existence of an excess of electrical charges in the pore water, in the so-called diffuse layer coating the surface of the minerals. In other words, the pore water is not neutral and the advection of its excess of charges denotes a source current density. In Eq. (5), φ is the electrical potential (in V, streaming-potential field, also called the self-potential field), $\mathbf{E} = -\nabla \varphi$ is the electrical field (V m^{-1}) (electromagnetic induction is neglected at very low frequencies and $\nabla \times \mathbf{E} = 0$), and σ is the electrical conductivity of the porous material (Sm^{-1}), The source current density of electrokinetic origin in Eq. (5) is given by $\mathbf{j}_S = \hat{Q}_\nu \mathbf{u}$ where $\hat{Q}_\nu$ (Cm^{-3}) is the effective excess of electrical charges of the diffusive layer per unit pore volume that is dragged by the flow field (see discussion in [41,44]), and $\mathbf{u}$ (ms^{-1}) is the Darcy velocity defined as $\mathbf{u} = -K\nabla h$ (Darcy's law). This excess of electrical charge is defined in a dynamic sense using ([48], their Appendix A),

$$\hat{Q}_V = \frac{\langle \rho \dot{\mathbf{u}}_f \rangle}{\langle \dot{\mathbf{u}}_f \rangle}. \tag{6}$$

where ρ (Cm^{-3}) is the local charge density in the water-saturated pore space, $\dot{\mathbf{u}}_f$ is the relative fluid veocity with respect to the solid phase, and the brackets denotes a volume average. Jardani et al. [26] formulated an empirical relationship connecting the effective or dynamic (excess) charge density $\hat{Q}_\nu$ and the hydraulic conductivity K given by $\log_{10}\hat{Q}_\nu = -3.49 - 0.82\log_{10}K$. This relationship has been then validated in unsaturated conditions and validated for a range of lithologies [48]. There a small dependence of $\hat{Q}_\nu$ with the salinity of the pore water, which has been discussed extensively in Ikard et al. [22]. This effect can be generally neglected because the strongest salinity effect is through the dependence of the conductivity σ with the salinity.

In the absence of external current sources and in the quasi-static limit of the Maxwell equations, the continuity equation for the electrical charges is [52]

$$\nabla \cdot \mathbf{j} = 0. \tag{7}$$