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Critical hydraulic gradient for nonlinear flow through rock fracture networks: The roles of aperture, surface roughness, and number of intersections

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ABSTRACT

Transition of fluid flow from the linear to the nonlinear regime has been confirmed in single rock fractures when the Reynolds number (*Re*) exceeds some critical values, yet the criterion for such a transition in discrete fracture networks (DFNs) has received little attention. This study conducted flow tests on crossed fracture models with a single intersection and performed numerical simulations on fluid flow through DFNs of various geometric characteristics. The roles of aperture, surface roughness, and number of intersections of fractures on the variation of the critical hydraulic gradient (J_c) for the onset of nonlinear flow through DFNs were systematically investigated. The results showed that the relationship between hydraulic gradient (J) and flow rate can be well quantified by Forchheimer's law; when J drops below J_c, it reduces to the widely used cubic law, by diminishing the nonlinear term. Larger apertures, rougher fracture surfaces, and a greater number of intersections in a DFN would result in the onset of nonlinear flow at a lower J_c. Mathematical expressions of J_c and the coefficients involved in Forchheimer's law were developed based on multi-variable regressions of simulation results, which can help to choose proper governing equations when solving problems associated with fluid flow in fracture networks.

1. Introduction

In recent years, discrete fracture network (DFN) modeling techniques have been extensively applied to investigations of hydro-mechanical and mass transport behaviors of fractured rock systems [36,38]. Many studies presumed that fluid flow in each single fracture in DFNs follows the cubic law, which suggests a linear relationship between the flow rate and the pressure drop in the calculations of fluid flow in fracture systems [27,39,41,43]. However, accurate estimations by the cubic law that neglects the inertial effects can only be anticipated for sufficiently low Reynolds numbers (*Re*), and past studies have revealed that the flow rate could be nonlinearly related to the pressure drop when the applied pressure/flow rate becomes large [14,20,22,23,30,32,65,68].

Previous studies have focused on the determination of the critical *Re* for the onset of nonlinear flow through single rock fractures, which suggested different ranges from 0.001 to 100 that vary with the surface roughness of fractures and applied normal and/or shear stresses [11,25,35,51,56,66,67,76,77]. The surface roughness that contributes to complex void geometries and streamline structures can significantly reduce the critical *Re* for the flow regime transition [24,33,53,58,63,76]. Studies on fluid flow through crossed fractures revealed complex nonlinear flow patterns when $1 < Re < 100$ and different mixing behaviors, bounded by complete mixing and streamline routing within fracture intersections [34,44,60].

The *Re* was typically incorporated in some criteria for detecting the nonlinear flow through single fractures, which may not apply to DFNs, because flow in each single fracture can have a different *Re*, and the assessment of the values of localized *Re* in DFNs with large amounts of fractures would be a tough task. In contrast, the hydraulic gradient (J), defined as the ratio of hydraulic head difference to DFN side length, is typically a known parameter in many practices on fractured rock masses, such as hydraulic pumping tests with prescribed hydraulic pressures. For a single fracture, the magnitude of *Re* is proportional to that of J, and J is also a dimensionless parameter representing how fast a pressure drops over a given region. Therefore, J may be a more practical parameter for establishing a criterion for the onset of nonlinear flow in DFNs.

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Although the mechanisms, such as the formation of vortices in the positions with dramatic geometric variations at a large *Re* that drive the nonlinear flow in rough-surfaced fractures and fracture intersections, have been extensively investigated [e.g., [12,34,35,76]], the impacts of these micro-phenomena on macro hydraulic properties of DFNs have not been quantitatively estimated. Due to the enormous difficulties of establishing DFN models to consider the roughness of each single fracture and of solving the Navier–Stokes (NS) equations composed of a set of coupled nonlinear partial derivatives of varying orders [10,24,77], most previous works presumed that the cubic law was always applicable, disregarding the magnitude of J, such as $J = 1$ [31,40,41,71,72,74,75], $J = 0.1$ [70], $J = 0.001$ [2,3], and $J =$ unknown constants [15,37,43,52,59,73]. It is therefore a crucial issue to determine the critical hydraulic gradient (J_c) for the onset of nonlinear flow in DFNs, below which the widely used cubic law is sufficiently applicable, and above which some nonlinear governing equations (e.g., Forchheimer's law) need to be employed.

In this study, we conducted flow tests on two crossed fracture models with a single intersection and performed their numerical simulations by solving the NS equations, which are fundamental elements in DFNs. After understanding the nonlinear fluid flow behavior through single intersections, we then established a series of DFNs with different apertures, roughness, and numbers of intersections. Based on multi-variable regressions of their simulation results, mathematical expressions of J_c and the coefficients involved in Forchheimer's law (A and B) were established. These expressions were applied to another series of DFNs with well-known geometric characteristics of fractures to verify their validity by comparing the predicted results with the fluid flow simulation results, and their nonlinear flow behaviors were analyzed and discussed.

2. Crossed fracture models with a single intersection

2.1. Experimental setup

2.1.1. Specimen preparation

Two fractured glass plates that contained smooth and rough-walled fractures respectively were utilized to establish two crossed fracture models in which two fractures intersected at an angle of 60°, as shown in Fig. 1(a). Fractures in the smooth-walled model were manufactured using a cutting machine, so that the two walls of each fracture are strictly parallel to each other with smooth surfaces. These fractures formed the parallel-plate model in which laminar flow follows the cubic law. Fractures in the rough-walled model were also manufactured using the cutting machine based on a pre-designed fracture sketch, in which the surface roughness of fractures was randomly generated. Since glass is a kind of brittle material, unexpected damages happened during the cutting that formed several unmated parts between the two walls of each fracture. This made the geometry of generated model different from the pre-designed sketch, which needed to be precisely measured as presented in the following sections. These unmated fractures are common in nature especially in shallow rock masses where dissolution of minerals along with ground water flow is active [61]. After the cutting, the walls of fractures are perpendicular to the x–y plane, except for several damaged parts. These models containing a single intersection were considered to be a starting point for investigations of nonlinear flow behavior of DFNs, which are usually composed of a great number of fractures and intersections. The side length of the models is 50 cm and the thickness is 5 mm. Another two glasses were firmly glued to the top and bottom faces (x–y plane) of the fractured glass to seal a model. One inflow tank was installed to guide fluid from a syringe pump to the model and three outflow tanks were installed to guide the fluid from the model to weighting containers. These tanks could help maintain steady flow before importing fluid into the fractures, and the pressure gauges were connected to these tanks to measure the differential pressure between the inlet and outlet. The role of these tanks could be switched to generate different combinations of inflow and outflow, such as one inlet and one outlet, one inlet and two outlets, and two inlets and two outlets.

2.1.2. Aperture estimation

The distance (aperture) between two walls of each fracture was fixed to a value of 1 mm when manufacturing the models. However, this process was implemented by hand using ordinary rulers, so that considerably large errors of magnitude of the apertures may be produced. To accurately estimate their apertures, a high-resolution CCD (charged coupled device) camera was utilized to take photographs of the two models filled with dye solutions (see Fig. 1(b)). The captured images were transformed into binary data, which were subsequently analyzed through image processing to obtain the x–y position of each point on the fracture surface. These images were then imported into AutoCAD, and polylines with a segment length of 0.01 mm were drawn upon the digital images to reproduce the fracture geometries (see Fig. 1(c)). The fracture geometry shown in Fig. 1(a) was obtained from the image processing of the actual physical model (Fig. 1(c)), instead of the pre-designed sketch. The mechanical apertures for the model with smooth fracture surfaces (smooth model: S) were $h_{M}\_1 = 1.37$ mm, $h_{M}\_2 = 1.40$ mm, $h_{M}\_3 = 1.20$ mm, and $h_{M}\_4 = 0.95$ mm; and for the model with rough fracture surfaces (rough model: R), the mechanical apertures were $h_{M}\_1 = 0.94$ mm, $h_{M}\_2 = 1.33$ mm, $h_{M}\_3 = 1.41$ mm, and $h_{M}\_4 = 0.98$ mm. The mechanical aperture (h_M) is defined as the arithmetic average of the point-to-point distance between the two walls of a fracture in the x–y plane. Measurements made at different hydraulic pressures showed that the mechanical aperture did not change with the changing hydraulic pressure.

2.1.3. Roughness estimation

The surface roughness of each fracture in the rough model was quantified by a widely used dimensionless parameter, Z_2, that ranges from 0.14 to 0.43, corresponding to the range of the joint roughness coefficient (JRC) [6] of 4.47–20.30, as shown in Fig. 1(a). Since these fractures are partially unmated, their two walls have slightly different values of Z_2. The values of Z_2 and JRC were calculated by the following equations [46,62]:

$$Z_2 = \left[\frac{1}{M} \sum \left(\frac{z_{i-1} - z_i}{x_{i-1} - x_i} \right)^2 \right]^{1/2}, \tag{1}$$

$$\mathrm{JRC} = 32.2 + 32.47 \log Z_2, \tag{2}$$

where x_i and z_i represent the coordinates of the fracture surface profile, and M is the number of sampling points along the length (i.e., x coordinate) of a fracture. Notable is that glass tends to form conchoidal fracture surfaces that lack micro-roughness; therefore, our models were designed to represent the first-order roughness of fractures formed by primary asperities, which play dominant roles in the nonlinear fluid flow properties of fractures [78]. Each fracture in the smooth model has a constant aperture and smooth surfaces with the value of Z_2 approximately 0.

2.1.4. Testing system and experimental procedure

A schematic view of the flow testing system is shown in Fig. 2. Fluid flow was introduced into the model from one inlet via a high precision syringe pump with an accuracy of ± 0.001 mL/min, and the effluents at other outlets were collected and measured by electronic balances with an accuracy of ± 0.01 g. The pressure drop between the inlet and each outlet was measured by a differential pressure gauge with an accuracy of ± 10 Pa. The differential pressure gauge measures the pressure difference between the two points (i.e., the inlet and outlet) in a system via tubes. The pressure at the exit of