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Hybrid upwind discretization of nonlinear two-phase flow with gravity

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ABSTRACT

Multiphase flow in porous media is described by coupled nonlinear mass conservation laws. For immiscible Darcy flow of multiple fluid phases, whereby capillary effects are negligible, the transport equations in the presence of viscous and buoyancy forces are highly nonlinear and hyperbolic. Numerical simulation of multiphase flow processes in heterogeneous formations requires the development of discretization and solution schemes that are able to handle the complex nonlinear dynamics, especially of the saturation evolution, in a reliable and computationally efficient manner. In reservoir simulation practice, single-point upwinding of the flux across an interface between two control volumes (cells) is performed for each fluid phase, whereby the upstream direction is based on the gradient of the phase-potential (pressure plus gravity head). This upwinding scheme, which we refer to as Phase-Potential Upwinding (PPU), is combined with implicit (backward-Euler) time discretization to obtain a Fully Implicit Method (FIM). Even though FIM suffers from numerical dispersion effects, it is widely used in practice. This is because of its unconditional stability and because it yields conservative, monotone numerical solutions. However, FIM is not unconditionally convergent. The convergence difficulties are particularly pronounced when the different immiscible fluid phases switch between co-current and counter-current states as a function of time, or (Newton) iteration. Whether the multiphase flow across an interface (between two control-volumes) is co-current, or counter-current, depends on the local balance between the viscous and buoyancy forces, and how the balance evolves in time. The sensitivity of PPU to small changes in the (local) pressure distribution exacerbates the problem. The common strategy to deal with these difficulties is to cut the timestep and try again. Here, we propose a Hybrid-Upwinding (HU) scheme for the phase fluxes, then HU is combined with implicit time discretization to yield a fully implicit method. In the HU scheme, the phase flux is divided into two parts based on the driving force. The viscous-driven and buoyancy-driven phase fluxes are upwinded differently. Specifically, the viscous flux, which is always co-current, is upwinded based on the direction of the total-velocity. The buoyancy-driven flux across an interface is always counter-current and is upwinded such that the heavier fluid goes downward and the lighter fluid goes upward. We analyze the properties of the Implicit Hybrid Upwinding (IHU) scheme. It is shown that IHU is locally conservative and produces monotone, physically-consistent numerical solutions. The IHU solutions show numerical diffusion levels that are slightly higher than those for standard FIM (i.e., implicit PPU). The primary advantage of the IHU scheme is that the numerical overall-flux of a fluid phase remains continuous and differentiable as the flow regime changes between co-current and counter-current conditions. This is in contrast to the standard phase-potential upwinding scheme, in which the overall fractional-flow (flux) function is non-differentiable across the boundary between co-current and counter-current flows.

1. Introduction

In reservoir simulation applications, including oil/gas recovery processes, groundwater remediation, and CO_2 subsurface sequestration, a Fully Implicit Method (FIM) [2] is widely used to solve the coupled nonlinear mass conservation equations. The standard FIM scheme employs implicit (backward-Euler) discretization for time and Phase-Potential-based Upwinding (PPU) for space. For the PPU scheme, the upstream direction of a fluid-phase is determined based on the gradient of its potential (i.e., phase pressure plus gravity head) across the interface between two control volumes (cells) [13,14,3,7,5]. In a simulation model,

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the number of mass conservation equations per control-volume is equal to the number of components used to describe the subsurface flow process. For immiscible two-phase flow, which is the focus of this work, there are two nonlinear conservation equations per computational cell (control volume). So, using the standard FIM (i.e., backward-Euler PPU) entails solving a system of coupled nonlinear algebraic equations (usually cast in residual form) for each timestep. A Newton-based strategy with full Jacobian matrices is the method of choice as the nonlinear solver. Even though the computational cost of repeatedly constructing and solving FIM Jacobian systems is substantial, the primary driver for the wide use of FIM in reservoir simulation is its unconditional stability [13,2]. Thus, in theory, FIM allows for taking arbitrary timestep sizes without worrying about the numerical stability of the computations. In practice, however, FIM is not guaranteed to converge for the chosen timestep, and we will discuss this issue further. Nevertheless, the favorable stability properties of FIM are highly desirable because the fully-implicit treatment can deal effectively with the different nonlinearities and the enormously wide range of (phase) velocities across the computational domain at a given time [9]. In addition to computational cost, the unconditional stability of FIM must be reconciled with the low-order numerical solutions, namely, first-order in space and time.

The convergence properties of FIM depend strongly on the physics being modeled, the spatial and temporal discretization schemes employed, and the nonlinear solution algorithm that are used. Nevertheless, it is safe to state that reservoir simulation using standard FIM (i.e., Implicit-PPU) is far from being unconditionally convergent [8,18]. In practice, nonlinear convergence problems in the course of a reservoir-simulation are a major concern because they can easily cause severe restrictions on the timestep size that can be used. To deal with such nonlinear convergence difficulties, reservoir simulators often employ a host of heuristics to adaptively control the timestep size, including timestep cutting. Different reservoir simulators and different users employ different heuristics, which are often tailored to a specific class of problem, and even a specific 'important' reservoir model. Overall, such convergence problems have a significant negative impact on the robustness of reservoir simulators, and they limit our ability to perform simulations (e.g., sensitivity studies and uncertainty quantification) of high-resolution reservoir-characterization models in a predictable manner. Moreover, as the need to model complex subsurface processes increases, the demands on the nonlinear solver are expected to increase dramatically.

Part of the challenge in developing physics-based nonlinear solvers is that the coupled nonlinear conservation equations governing multiphase flow and transport in heterogeneous porous media are quite difficult to analyze. As a result, there has been a tradition of analyzing linearized subproblems (e.g., flow, or transport) often for mixed-implicit formulations, instead of dealing directly with FIM formulations. So, when it comes to the nonlinear solution strategy, the complex interactions between (1) the spatial and temporal scales that govern the physics of flow and transport, (2) the spatial and temporal discretization, and (3) the nonlinear and linear solvers have received little attention. The result is that, in practice, the task of solving large nonlinear systems of coupled algebraic equations for a given timestep is tackled using a damped (safeguarded) Newton method (usually with full Jacobians) guided by various heuristics to detect convergence problems and prescribe a remedy, including cutting the timestep and restarting the iterative process.

Jenny et al. [8] developed a physics-specific nonlinear solver for the saturation (transport) equation of immiscible two-phase flow without buoyancy effects. They used the inflection point of the analytical flux (fractional-flow) function to guide the Newton-based iterative updating of the saturation field. Jenny et al. embedded their saturation (transport) nonlinear solver into a Sequential Fully Implicit (SFI) scheme. In their SFI strategy, the pressure is solved implicitly, and the total-velocity field is computed; then, the saturation is obtained using their inflection-based Newton solver. An outer loop is wrapped around the two sequential loops of implicit-pressure and implicit-saturation. Their scheme was proved to be unconditionally convergent for immiscible two-phase flow without buoyancy. The proof relied on the use of potential-ordering [9] of the FIM system to split the flow (pressure and total-velocity) from the transport (saturation). The saturation is then updated cell-wise based on the properties of the analytical flux function. Specifically, the nonconvex, but monotone, fractional-flow (flux) function was divided into two trust regions delineated by the inflection point. If the local (control-volume) saturation update obtained by the full Newton update would cross the inflection saturation, the update is simply chopped back to the inflection point, and the iterative process is continued.

When in addition to the viscous forces, buoyancy and/or capillary forces are present, numerical simulation of multiphase flow gets much more complicated. In this paper, we focus on immiscible two-phase flow in the presence of both viscous and buoyancy forces, and we ignore the effects of capillarity. As the relative importance of buoyancy increases, the analytical fractional-flow (phase-flux/total-flux) function can become nonmonotonic; this function usually has two inflection points. The details of nonmonotonicity and nonconvexity of the (analytical) flux function depend on the relative-permeability relations, viscosity ratio, and the (local) balance of the viscous and buoyancy forces. The nonmonotonicity of the analytical fractional-flow is associated with counter-current flow, whereby the two immiscible fluids flow in opposite directions across the interface of interest.

Many nonlinear convergence problems of the standard FIM (i.e., Implicit-PPU) formulation have been linked to the occurrence of counter-current flow - as a function of space and time. In multiphase problems, changes between co-current and counter-current flow regimes as a function of time, or (Newton) iteration, can lead to oscillations in the computed saturations and ultimate divergence of the solver. In the standard Phase-Potential-based Upwinding (PPU) spatial discretization scheme [13,2,3], the upstream direction of a phase-flux is determined based on the overall phase-potential (phase-pressure plus gravity head) difference. So, as the phase-flux changes between co-current and counter-current states, the upstream direction for the fluid-phase gets switched accordingly, and the numerical fractional-flow function becomes discontinuous. In most reservoir models, neighboring cells often have different properties, especially for the absolute permeability, and the spatial distribution and temporal evolution of the flow regimes (co-current and counter-current) in the computational domain can be quite complex. The discontinuities in the PPU numerical flux across the co-current/counter-current flow regimes can be quite severe, and they can lead to ill-conditioned Jacobian matrices and serious convergence problems in the Newton-based iterative solution process [11].

Wang and Tchelepi [17] extended the work of Jenny et al. [8] to nonlinear two-phase problems in the presence of both viscous and buoyancy forces. They proposed a 'trust-region' Newton solver, in which the analytical fractional-flow function is divided into trust regions of saturation. In simple terms, a 'trust region' is one for which the Newton method is guaranteed to converge. The saturation intervals were delineated by the end-points, inflection points, and the unit-flux point of the analytical fractional-flow. Wang and Tchelepi used their 'trust-region' modified-Newton nonlinear scheme to improve the robustness and efficiency of the standard FIM (i.e., implicit PPU) approach for problems with gravity. They showed excellent convergence behavior for highly heterogeneous three-dimensional reservoir models in the presence of viscous