Advances in Water Resources 79 (2015) 18–34

Contents lists available at ScienceDirect

Advances in Water Resources

journal homepage: www.elsevier.com/locate/advwatres

A multi-scale turbulent dispersion model for dilute flows with suspended sediment

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ARTICLE INFO

Article history:
Received 21 December 2013
Received in revised form 3 February 2015
Accepted 3 February 2015
Available online 24 February 2015

Keywords:
Turbulent dispersion
Suspended sediment
Two phase flow

ABSTRACT

Turbulent dispersion causes sediment particles to be transported from high concentration regions to low concentration regions and determines the concentration distribution of suspended sediment. In this study, a new turbulent dispersion model is proposed for large-scale flows with suspended sediment. Two Stokes numbers are used to describe the turbulent dispersion through the Schmidt number: (1) $st \equiv \tau_p/\tau_f$, where τ_p is the particle response time and τ_f the fluid turbulence time scale, and (2) $st_\eta \equiv \tau_p/\tau_\eta$, where τ_η is the Kolmogorov time scale. The former is used to account for interaction between large eddies and sediment particles, while the latter for small eddies. The new turbulent dispersion model is validated against experimental data available for open channel flows under a wide range of conditions *for dilute flows*: the volume concentration of suspended sediment could vary from 10^{-6} to 0.08, st could vary from 2×10^{-3} to 8 and st_η could vary from 2×10^{-2} to 60.

1. Introduction

For sediment transport in turbulent dispersed multiphase flows, the fluid is the carrier phase and the sediment is the dispersed phase. Interactions between turbulent eddies and sediment particles exhibit different characteristics at different values of Stokes number st, which is defined as $st \equiv \tau_p/\tau_f$ with τ_p being a particle response time scale related to the drag force and τ_f a time scale for turbulent eddies of the fluid phase [1,2]. When $st << 1$, sediment particles have very short response times and behave more like fluid particles; when $st \approx 1$, although sediment particles no longer follow fluid motion well, the trajectories of sediment particles can still be influenced significantly by turbulence; when $st >> 1$, the turbulence effect on particle motion becomes less significant because of the large inertia of the sediment particles [3]. Complex interactions between turbulent eddies and sediment particles lead to turbulent dispersion [4–6] and turbulence modulation [2,7,8]. Turbulence modulation refers to the influence of the sediment phase on the turbulence of the carrier phase. Turbulent dispersion causes sediment particles to be transported from high concentration to low concentration regions, and it is a key factor in determining the concentration distribution of suspended sediment.

Two classes of approaches are available for modeling flows with suspended sediment: single-phase flow approach [9–11] and two-phase flow approach [12–16]. A single-phase flow approach regards sediment as a passive scalar, and the turbulent dispersion is described by $\overline{c'u'_i}$ in the mass balance equation, where c' represents the fluctuating concentration of sediment and u'_i (with i = 1, 2, and 3) the fluctuating velocity of the carrier phase, and the overbar stands for a time average operation. The turbulent dispersion term ($\overline{c'u'_i}$) is further modeled by applying the gradient-transport hypotheses [17], i.e. $\overline{c'u'_i} = -(\nu_{ft}/\sigma_d)\partial\bar{c}/\partial x_i$ where ν_{ft} is the eddy viscosity, σ_d is the Schmidt number, $\bar{c}$ is the mean concentration of sediment, and x_i represents the ith spatial coordinate. A single-phase flow approach can only consider the effects of flow field on the transport of the passive scalar. A two-phase flow approach, on the other hand, considers the fluid and sediment as two continuous materials that occupy the same point in space simultaneously [18]. In two-phase flow approaches, the macroscopic motions of fluid and sediment phases are governed by their own equations, which are obtained by averaging the microscopic governing equations for each phase [19], and the effects of sediment–fluid interaction can be considered. However, the averaging operation does not explicitly introduce any turbulent

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http://dx.doi.org/10.1016/j.advwatres.2015.02.002
0309-1708/

dispersion term in the resulting equations. There are three approaches available to introduce turbulent dispersion into the two-phase flow models describing the macroscopic motions of two-phase flows.

In the first approach, macroscopic motions of fluid and sediment phases include turbulent fluctuations, and another averaging operation is needed to filter out the turbulent components (see, e.g. Ref. [16]). The final equations governing the macroscopic mean motions of fluid and sediment phases, obtained by performing two separate averaging operations, include a turbulent dispersion term in the mass-balance equation (see, e.g. Ref. [16]). The turbulent dispersion term, which is the same as that in the single-phase approach ($\overline{c'u'_i}$), is modeled by using the adopted gradient-transport hypotheses [17]. The main difficulty that the first approach may face is that the sediment phase may have non-zero flux even when the macroscopic mean velocity for sediment phase is zero, which is physically inconsistent [20].

The second approach is similar to the first, except that the second averaging operation is concentration-weighted (Favre average) so that the terms related to $\overline{c'u'_i}$ do not appear in the mass-balance equation. This approach considers the fluctuating interfacial force to be responsible for turbulent dispersion [6], and introduces the turbulent dispersion term $\rho^s\overline{c'u'_i}/\tau_p$ in the momentum equations, where ρ^s is the density of sediment (see, e.g. Refs. [13,21]). This approach still uses the gradient-transport hypotheses [17] to model the turbulent dispersion term, i.e. $\rho^s\overline{c'u'_i}/\tau_p = -\rho^s(\nu_{ft}/\sigma_d)\partial\bar{c}/\partial x_i/\tau_p$.

In the third approach, the macroscopic motions of both the sediment phase and fluid phase, obtained by averaging the microscopic governing equations for each phase, do not include turbulent fluctuations. Turbulent dispersion is considered in the momentum equations by using probability-density-function-based (PDF-based) approaches to analyze the fluctuating interfacial force (see, e.g. Ref. [22]).

The second and third approaches give the same expression for the turbulent dispersion term, i.e., $-\rho^s(\nu_{ft}/\sigma_d)\partial\bar{c}/\partial x_i/\tau_p$; however, these two approaches use different k–ε equations to model the eddy viscosity ν_{ft} [8].

In all these approaches, a key issue is how to model the Schmidt number σ_d. However, there is currently no unified model for the Schmidt number, and inconsistencies sometimes exist among various models. Physically, the Schmidt number represents the ratio of the eddy viscosity of the fluid phase to the eddy diffusivity of the sediment phase, and its value strongly affects the distribution of sediment concentration. Some previous studies attempted to quantify σ_d. For example, van Rijn [23] stated that σ_d should be less than 1 because of the centrifugal forces acting on sediment particles and that σ_d should decrease with increasing ratio of the fall velocity of the sediment particle to the shear velocity at the bottom. Jha and Bombardelli [16] indicated that σ_d could be larger than 1. However, the Schmidt number was treated as a fitting parameter in Jha and Bombardelli [16]. Lees [24] found from their experiments that σ_d increased with increasing sediment concentration. Amoudry et al. [25] proposed a formula to relate σ_d to sediment concentration by comparing their model predictions with experimental data [26], but the parameters are not universal [16]. Direct numerical simulations for heavy particles in a turbulent flow [1] found that σ_d should depend on Stokes number *st*. Some studies [22,27] based on PDF-based approaches (see, e.g. [28–30]) also found that σ_d should depend on *st*: de Bertodano [22] stated that σ_d should increase linearly with *st*; Fu et al. [27] stated that σ_d depends on *st*, and found that σ_d decreased toward the bed for open-channel flows with suspended sediment, which is contrary to the results of Amoudry et al. [25]. Clearly, there is a need to further study the relationship between *st* and σ_d for two-phase flows with suspended sediment.

This study aims to model the turbulent dispersion for large-scale problems involving flows with suspended sediment by considering turbulence–particle interactions under a two-phase-flow framework. We adopt in this study the approach in which averaging the microscopic governing equations will filter out turbulent fluctuations, i.e., the third approach mentioned above. By analyzing σ_d for different values of *st*, a new turbulent-dispersion model is introduced that considers the effects of both large and small eddies. In contrast to other studies, the proposed turbulent dispersion model considers the effect of multi-scale turbulence–particle interaction on σ_d. In addition to modeling turbulent dispersion, macroscopic stresses, interfacial forces between the carrier and dispersed phases and other derived effects such as turbulence modulation are modeled, and are done in this study by adopting the rigorous microscopic definitions of Hwang and Shen [31–33] for macroscopic stresses, interfacial forces and fluctuating energy equations. The resulting two-phase flow model is validated by comparing with published experiment data [9,34,35]. The dependence of σ_d on multi-scale turbulence–particle interaction for open channel flows with suspended sediment is also discussed.

The rest of the paper is organized as follows: Governing equations and closures for stresses are summarized in Section 2; the proposed turbulent dispersion model is introduced in Section 3 and verified in Section 4; effects of large and small eddies on the Schmidt number are discussed in Section 5, and main conclusions are given in Section 6.

2. Governing equations and closures for fluid and sediment stresses

This section presents the governing equations and models for fluid and sediment stresses. Models for turbulent dispersion and turbulence modulation will be discussed in Section 3.

2.1. Governing equations

Equations governing two-phase flows can be obtained by taking an ensemble average of the governing equations for each phase [36]. In order to identify which phase is present at a specific location at time, t, the following phase function is introduced in the averaging process [19]

$$c(x_1,x_2,x_3,t) = \begin{cases} 0, & \text{for fluid phase} \\ 1, & \text{for solid phase} \end{cases} \tag{1}$$

Using the phase function given in Eq. (1), the volumetric concentration of the sediment phase is defined by the ensemble average of c, denoted by $\langle c\rangle$ with $\langle\cdots\rangle$ denoting an ensemble average. The c-weighted average or phasic average of any physical property ϕ can be defined by $\{\phi\} = \langle c\phi\rangle/\langle c\rangle$ for the sediment phase and $\{\phi\} = \langle(1-c)\phi\rangle/\langle 1-c\rangle$ for the fluid phase (see, e.g. Ref. [19]).

This study focuses on the interaction between sediment grains and water, and there is no mass transfer between these two phases. Accordingly, the ensemble-averaged governing equations of mass and momentum for the fluid and sediment phases have the following forms [36]:

For the fluid phase,

$$\frac{\partial\rho^f(1-\langle c\rangle)}{\partial t} + \frac{\partial\rho^f(1-\langle c\rangle)\{u_i\}}{\partial x_i} = 0 \tag{2}$$

$$\frac{\partial\rho^f(1-\langle c\rangle)\{u_i\}}{\partial t} + \frac{\partial\rho^f(1-\langle c\rangle)\{u_i\}\{u_j\}}{\partial x_j} = \rho^f(1-\langle c\rangle)g_i - \langle m_i\rangle + \frac{\partial(1-\langle c\rangle)\{T^f_{ji}\}}{\partial x_j} \tag{3}$$