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Combined effect of rheology and confining boundaries on spreading of gravity currents in porous media

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ABSTRACT

One-dimensional flows of gravity currents within horizontal and inclined porous channels are investigated combining theoretical and experimental analysis to evaluate the joint effects of channel shape and fluid rheology. The parameter β governs the shape of the channel cross section, while the fluid rheology is characterised by a power-law model with behaviour index n. Self-similar scalings for current length and height are obtained for horizontal and inclined channels when the current volume increases with time as t^{α}.

For horizontal channels, the interplay of model parameters α, n, and β governs the front speed, height, and aspect ratio of the current (ratio between the average height and the length). The dependency is modulated by two critical values of $\alpha, \alpha_\beta = n/(n+1)$ and $\alpha_n = (2\beta+1)/\beta$. For all channel shapes, α_β discriminates between currents whose height decreases ($\alpha < \alpha_\beta$) or increases ($\alpha > \alpha_\beta$) with time at a particular point. For all power-law fluids, α_n discriminates between decelerated currents, with time-decreasing aspect ratio ($\alpha < \alpha_n$), and accelerated currents, with time-increasing aspect ratio ($\alpha > \alpha_n$). Only currents with time-decreasing height ($\alpha < \alpha_\beta$) and aspect ratio ($\alpha < \alpha_n$) respect model assumptions asymptotically; the former constraint is more restrictive than the latter.

For inclined channels, a numerical solution in self-similar form is obtained under the hypothesis that the product of the channel inclination θ and the slope of the free-surface is much smaller than unity; this produces a negligible error for $\theta > 2^\circ$, and is acceptable for $\theta > 0.5^\circ$. The action of gravity in inclined channels is modulated by both the behaviour index n and the shape factor β. For constant flux, the current reaches at long times a steady state condition with a uniform thickness profile. In steep channels and for sufficiently long currents, the free-surface slope becomes entirely negligible with respect to channel inclination, and the constant thickness profile depends only on n.

Theoretical results are validated by comparison with experiments (i) in horizontal and inclined channels with triangular or semicircular cross-section, (ii) with different shear-thinning fluids, and (iii) for constant volume and constant flux conditions. The experimental results show good agreement with theoretical predictions in the long-time regime.

Our analysis demonstrates that self-similar solutions are able to capture the essential long-term behaviour of gravity currents in porous media, accounting for diverse effects such as non-Newtonian rheology, presence of boundaries, and channel inclination. This provides a relatively simple framework for sensitivity analysis, and a convenient benchmark for numerical studies.

1. Introduction

Gravity currents are responsible for many natural processes occurring in the atmosphere, water bodies and the subsurface, as whenever two fluids of different density come in contact, the gravity action favours their relative motion. The interest in gravity currents has generated a vast literature, including theoretical, numerical and experimental studies (e.g. [1,2] and references therein). Gravity currents in porous media involve the spreading of a fluid in a natural or artificial domain saturated with another fluid of a different density; the pressure/buoyancy driving is balanced by viscous adjustment of the fluid in the pore space. This phenomenon has been studied in connection with environmental and industrial applications such as aquifer remediation [3,4], carbon sequestration [5], saltwater intrusion [6], and well drilling [7].

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Earlier studies [8–10] have addressed the one-directional propagation of gravity currents driven by gradients in hydrostatic pressure (horizontal bed) or gravity (inclined bed) in homogeneous domains with simple geometry. Further complexity in the flow description arises when either heterogeneity in medium properties or topographic control is present in the flow domain, as suggested by diverse applicative contexts, such as modelling of carbon dioxide injection in natural reservoirs [11].

Within medium heterogeneity, Ciriello et al. [12] considered the influence of vertical and horizontal permeability variations on the propagation of planar currents. Zheng et al. [13,14] further extended the study of gravity driven flows in elongated porous domains, by considering: (i) coupled permeability and porosity gradients parallel and transverse to the flow direction; (ii) currents propagating away or towards the domain origin, giving rise to different types of self-similar behaviour. These studies showed that macro-heterogeneity in medium properties alters both the extension and the shape of the intruding current.

As for topographic control, Golding and Huppert [15] investigated the effect of confining boundaries on one-dimensional gravity-driven flow in porous channels, by considering a uniform cross-section with shape described by one parameter. The propagation rate is affected by channel shape in a way depending on the time exponent of the current volume; when the channel has a slope much steeper than its free surface, the spreading rate is unaffected by confining boundaries. Pegler et al. [16] analysed the effect of an upward sloping topography in the flow direction on constant flux currents. Topography was shown to control the early or late-time evolution of the current, depending on the shape of the lower boundary over which the current flows.

The rheology of the intruding fluid is another key factor in controlling the propagation of gravity currents in porous media [17,18]. In some cases the ambient and current fluids may be appropriately described as Newtonian, but in many instances, one or both fluids behave as non-Newtonian; relevant cases include polymer solutions, heavy oils, surfactants, foams, gels, emulsions, greases, and water-based slurries used for aquifer remediation [19]. The rheology of non-Newtonian fluids of interest in porous media flow encompasses several possible models [20]; among these, the power-law model provides the simplest relationship between stress and strain, and constitutes an acceptable approximation when: (i) the fluid is purely viscous; (ii) yield stresses are negligible, (iii) no other physical effects, such as adsorption, are present; (iv) the fluid rheologic parameters are evaluated in the range of shear rates occurring in the medium at the pore scale [21,22].

The impact of heterogeneity in porous medium properties is particularly relevant for non-Newtonian flow, as suggested by the numerical simulations of Fadili et al. [23], who showed that in correlated media streamlines of shear-thinning flow tend to concentrate along higher permeability paths. To explore the combined effect of fluid rheology and spatial heterogeneity, Di Federico et al. [24] derived a closed-form solution for radial gravity currents of power-law fluids in porous media with a deterministic permeability variation along the vertical; the solution was then validated experimentally. The axisymmetric scheme describes the motion of a gravity current originating from a single vertical borehole and spreading in an infinite domain. For currents propagating in narrow, elongated domains, topographic features may control the flow, yet to the best of our knowledge the effect of topographic control on non-Newtonian gravity currents has never been explored.

To this end, we firstly investigate in this work the combined effect of power-law rheology and confining boundaries on the motion of gravity currents in porous media. The boundary effect is represented by means of a channel of constant cross-section along the lines of Golding and Huppert [15], with the intruding fluid flowing (i) at the impervious bottom, for currents denser than the ambient fluid, or (ii) at the impervious cap, for currents lighter than the ambient fluid. In both cases, the confined porous channel can be horizontal or inclined. A limiting case of channels engraved in impervious boundaries is represented by fractures and very narrow cross-section channels. This scenario was analysed for low-Reynolds number flows by Takagi and Huppert [25,26] for Newtonian fluids, and by Longo et al. [27] for shear-thinning fluids. The presence of porous material inside these fractures strongly influences flow behaviour [26].

In the following sections we first present the theoretical model partially introduced by Ciriello et al. [28] for the limited case of horizontal channels and here extended to the inclined case. Both cases are discussed in detail, focusing on sensitivity to model parameters and range of applicability of the proposed formulation. Then an extended set of laboratory experiments is presented and discussed in order to validate the theoretical formulation under diverse possible scenarios, including: (i) different fluids; (ii) channels of different shape; (iii) horizontal or inclined channels; (iv) constant volume or constant flux injection. A set of conclusions closes the paper.

2. Problem formulation

Consider a non-Newtonian fluid with rheology described by the classical power-law model relating shear stress τ and shear rate $\dot{\gamma}, \tau = \widetilde{\mu}\dot{\gamma}|\dot{\gamma}|^{n-1}$, having parameters $\widetilde{\mu}$ (consistency index) and n (fluid behaviour index). This fluid, of density $\rho + \Delta\rho$, is released at the origin of a straight channel of uniform inclination θ as depicted in Fig. 1. The channel is filled with a homogeneous porous medium saturated with a lighter fluid of density ρ. Under this scenario, a gravity current is generated, advancing in a condition of vertical equilibrium with hydrostatic pressure distribution. The height of the current is much smaller than its length, with consequent negligible vertical velocities. Surface tension effects and mixing at fluids interface are also unimportant. The channel cross-section is symmetric and described by a power-law relationship: $b(y) = ra(y/a)^\beta$, where β is a shape parameter, a a length associated with the channel width, and r a dimensionless constant. For wide cross sections with $\beta > 1$, the current is taken to occupy only a small portion of the channel, so that $h \ll a$. Note that: (i) the case $\beta = 1$ corresponds to a triangular cross-section with $r = \cot\chi$, being 2χ the vertex angle; (ii) the case $\beta = 2$ approximates a semicircular cross-section as the height of the current is limited compared to the cross section radius $a/2$; (iii) the case $\beta \to \infty$ corresponds to a rectangular channel of half width a; as $h \ll a$, two-dimensional flow on a flat surface is recovered.

Our model relies on (i) local mass balance, (ii) a 1-D seepage formula extending Darcy's law to the case of non-Newtonian fluid flow (e.g. [17]). These two equations read, respectively

$$\phi\frac{\partial}{\partial t}\left(A_c h^{(\beta+1)/\beta}\right) + \frac{\partial}{\partial x}\left(u_x A_c h^{(\beta+1)/\beta}\right) = 0 \quad (1)$$

$$u_x(x,t) = -(\Lambda\Delta\rho g)^{1/n} k^{(1+n)/(2n)}\left(\frac{\partial h}{\partial x}\cos\theta - \sin\theta\right)^{1/n}, \quad (2)$$

where ϕ is the porosity, k the medium permeability, $A_c = \frac{2\beta}{\beta+1}\frac{a^{(\beta-1)/\beta}}{r^{1/\beta}}$, and $\Lambda = \frac{8^{(n+1)/2}}{2}\left(\frac{n}{3n+1}\right)^n\frac{\phi^{(n-1)/2}}{\widetilde{\mu}}$. Substituting (2) in (1) and introducing the natural velocity scale $v^* = \frac{(\Lambda\Delta\rho g)^{1/n}k^{(1+n)/(2n)}}{\phi}$ yields

$$\frac{\partial h^{F_1}}{\partial t} + v^*(\sin\theta)^{1/n}\frac{\partial}{\partial x}\left[h^{F_1}\left(1 - \cot\theta\frac{\partial h}{\partial x}\right)^{1/n}\right] = 0, \quad (3)$$