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Using a mechanical approach to quantify flow resistance by submerged, flexible vegetation – A revisit of Kouwen's approach

Li Chen [a,b,*], Kumud Acharya [b], Mark C. Stone [c]

[a] State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, Hohai University, Nanjing, Jiangsu 210098, China
[b] Division of Hydrologic Sciences, Desert Research Institute, Las Vegas, NV 89119, United States
[c] Department of Civil Engineering, University of New Mexico, Albuquerque, NM 87106, United States

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ABSTRACT

Vegetation-induced flow roughness can result in significant changes in stream hydraulics. This study revisits the well-known empirical equation for submerged flexible vegetation developed by Kouwen and collaborators, which describes the relationships between shear stress, flexural rigidity, and vegetation deflection. Theoretical analysis shows that the theories for the mechanics of large deflection cantilever beams can essentially explain this equation. The results show that for moderate to large deflection (the ratio of deflected height to original height $l/L < 0.85 - 0.9$) the theoretically derived relationships can be approximated with power-law equations, which have similar exponents to the Kouwen's equation and agree with its empirical relationships, which indicates the consistency of the underlying physics for the two approaches. Direct comparisons under given vegetation-height conditions also show a general agreement between the empirical and the theoretical equations. For small deflections, the theoretical results exhibit a more intuitive trend, which shows that the shear stress approaches zero at infinitesimal deflection. Additionally, theoretical analysis suggests a different non-dimensional parameter for vegetation mechanical properties and a better structure of the equation, which is expected to improve the estimation of vegetation-induced roughness. Finally, theoretical analysis indicates that even though the structure maintains, the specific relationship between vegetation bending and resistance is dependent on the flow velocity profile. Further development of these approaches need to take flow characteristics into consideration.

1. Introduction

Vegetation introduces additional resistance to streamflow and potentially changes stream hydrodynamics and flood inundation risks. Flow roughness caused by vegetation has been studied for several decades and various approaches have been developed. For submerged, grass-type vegetation, the equation proposed by Kouwen and collaborators is widely adopted. This equation was developed based on concepts in mechanics of material and fluid. Kouwen and Unny [1] proposed to use the quantity MEI to describe the flexural rigidity, or stiffness, of vegetation in a unit area, where M [−] represents the stem density defined by these authors as the number of stems in 1 m^2, E [kg m^{-1} s^{-2}] is Young's modulus of the plant, and I [m^4] is the area moment of inertia. Based on dimensional analysis, they proposed a non-dimensional variable $\left(\frac{MEI}{\gamma hS}\right)^{0.25}\Big/L$ and linked it to vegetation bending described by l/L, where γ is the specific gravity of water, h is the flow depth, S is the friction slope, and L and l are the original and deflected heights of the plant. In this equation, γhS is the flow shear stress that introduces the resistance into the equation. Through flume experiments, these authors determined an empirical relationship between these two variables. Kouwen and Li [2] reformulated this relationship and presented the following equation:

$$\left(\frac{MEI}{\gamma hS}\right)^{0.25}\Big/L = 3.4\left(\frac{l}{L}\right)^{0.63} \tag{1}$$

The concept of flexural rigidity and the resulting vegetation-bending resistance equations (Eq. (1) and its earlier version by Kouwen and Unny [1]) have been widely applied in the literature e.g. [3–10] for studying flexible-vegetation-induced flow roughness.

Although Kouwen's approach is an empirical relationship, it introduces mechanical concepts into the vegetation resistance study that allow for fully understanding the detailed mechanics of the interaction between fluid and flexible vegetation. A complete

* Corresponding author at: State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, Hohai University, Nanjing, Jiangsu 210098, China.

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view of this system should include not only the flow characteristics, but also the biomechanical properties and responses of the vegetation. Therefore, using these mechanical concepts may lead to better quantification to the coupled system and may result in new approaches for dynamic flow resistance that account for changing flow resistance with the continuous deformation of vegetation. After reviewing the development of this approach, we are interested in exploring the following question in regards to the fundamental mechanism of the approach: in view of the physical basis of Eq. (1), can this equation be explained with a more physically rigorous approach? Answering this question will help advance the development of physically based approaches for vegetation resistance.

2. Theoretical analysis

In this section, we discuss whether we can explain the Kouwen relationship (Eq. (1)) with mechanical theories. The main assumptions adopted in the development of Eq. (1) include: vegetation bending is caused by the drag force; drag force is the dominant component in the flow resistance when a large number of plant stems appear; and the total drag force is the sum of the drag force for each individual plant.

Taking a closer look at Eq. (1), we find that it describes the relationships between shear stress, vegetation elastic rigidity, and vegetation deflection. In densely vegetated areas, the shear stress is mainly caused by the drag force on vegetation [11]. Essentially, Eq. (1) determines the balance between bending force and deflection for vertically placed cantilever beams. Therefore, we can use a theoretical approach to address the problem.

For a homogeneous grass-type vegetation field, we assume that each individual plant behaves independently and similarly in a uniform flow field. Therefore, the total resistance induced by vegetation is simply the sum of drag forces on all individual plants (see Fig. 1 for a schematic). We assume that the wake effect on each plant is equal in a large homogeneously vegetated area and that it can be accounted for in a known uniform velocity field and drag coefficient distribution. Based on this assumption, the result for the entire vegetated area can be scaled up by multiplying the stem density to the single-stem result. Therefore we start from the single-plant bending problem that is described well by the large deflection cantilever beam theory, which relates the deflection of the plant to the external load, which is mainly the drag. Under the assumption that the beam material remains linearly elastic, the Euler–Bernoulli equation for bending reads [12,13]:

$$\frac{d\theta}{ds} = \frac{M_l(s)}{EI} \tag{2}$$

where θ is the angle of rotation of the deflection curve, s is the distance measured along the beam, M_l is the moment of the load, E is the modulus of elasticity of the material and I is the second moment of area of the beam cross section about the bending axis.

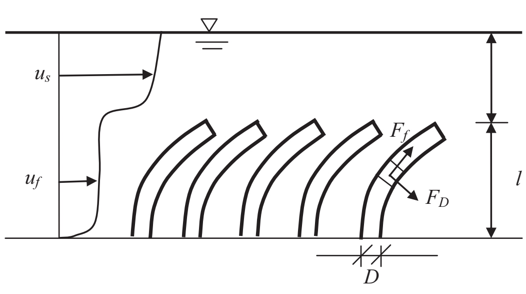

Fig. 1. A schematic view of a submerged plant in a real flow field.

Chen [14] converted this equation into the following form:

$$\frac{ds}{dy} = \frac{1}{\sqrt{1 - G^2(y)}} \tag{3}$$

where y is the vertical coordinate parallel to the undeflected beam, $G(y)$ is defined by:

$$G(y) = \int_0^y \frac{M_l(y)}{EI} dy \tag{4}$$

This is a general equation applicable for arbitrary loads and beam mechanical properties, and thus can be easily applied for theoretical analysis.

With simple load conditions, $G(y)$ can be explicitly formulated. For a uniformly distributed load, $M_l(y) = \frac{F_D}{l}(l-y)^2$, where F_D is the total load exerted over the deflected beam height l, and $G(y) = \frac{F_D/l}{2EI}\left(\frac{y^3}{3} - ly^2 + l^2 y\right)$. Eq. (3) can then be converted to a non-dimensional form:

$$\frac{ds_*}{dy_*} = \frac{1}{\sqrt{1 - \frac{r^2}{4}\left(\frac{y_*^3}{3l_*} - y_*^2 + l_* y_*\right)^2}} \tag{5}$$

B.C: $s_* = 1$ at $y_* = l_*$.

where parameter $r = \frac{F_D L^2}{EI}$, $l_* = l/L$, and L is the undeflected beam length. The parameter r describes the ratio of external load and beam rigidity.

Similarly, with a concentrated load F_D applied to the free end of the beam, Eq. (3) can be converted to:

$$\frac{ds_*}{dy_*} = \frac{1}{\sqrt{1 - r^2\left(l_* y_* - \frac{y_*^2}{2}\right)^2}} \tag{6}$$

Eqs. (5) and (6) suggest that the deflection of a cantilever beam with a load that is normal to the undeflected beam can be determined by a single parameter, r. Therefore, we can examine these non-dimensional equations to determine the relationship between vertical deflection l/L and the parameter r (i.e., $r = f(l/L)$, which is similar to Eq. (1)).

For a vegetation cluster, we can scale up the analysis for a single plant and express r using the bulk resistance and bulk rigidity. In a densely vegetated area, the flow resistance caused by shear stress at a solid boundary is negligible. The drag force dominates the total shear stress so that $\tau = \gamma hS \cong WF_D$, where the stem density W [L^{-2}] has the similar physical meaning to Kouwen's M [−], which, however, is a dimensionless parameter. Nevertheless, M is linked to the length scale (i.e., its value changes with different length units) whereas W is independent of it. This makes W a more suitable parameter for the analysis. Therefore, r also reads $r = \frac{\tau L^2}{WEI}$, in which WEI represents the vegetation rigidity in a unit area.

These two load distributions are limiting conditions for most real cases. In other words, the vertical distribution of the actual load on a plant in water flow usually lies between these two distributions. Therefore, we can examine the relationship between deflection and shear stress by solving these two cases.

The two ODEs (Eqs. (5) and (6)) can be solved with a straightforward search procedure [14]. For a given r, a value of the deflected height l_{*o} can be assumed and the equations can be numerically solved to obtain the entire bending curve, as well as the total deflection l_*. When l_* matches l_{*o}, the solution is found. We can search the value of l_* over [0,1] to determine l_* for the solution of the problem. The results are presented in Fig. 2, which show that different load distributions result in different relationships.

We can use empirical equations to approximate these curves for further analysis. A piecewise equation was found to be more suitable for these curves than a single power-law relationship. This can