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# A computational model for fluid leakage in heterogeneous layered porous media

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ABSTRACT

This paper introduces a new and computationally efficient model for the simulation of non-wetting phase leakage in a rigid heterogeneous layered medium domain constituting layers of different physical properties. Such a leakage exhibits a discontinuity in the saturation field at the interface between layers. The governing field equations are derived based on the averaging theory and solved numerically using a mixed finite element discretization scheme. This scheme entails solving different balance equations using different discretization techniques, which are tailored to accurately simulate the physical behavior of the primary state variables. A discontinuous non-wetting phase saturation–continuous water pressure formulation is adopted. The standard Galerkin finite element method is utilized to discretize the water phase pressure field, and the partition of unity finite element method is utilized to discretize the non-wetting phase saturation field. This mixed discretization scheme leads to a locally conservative system, giving accurate simulation of the saturation jump. The boundary between layers is embedded within the finite elements, alleviating the need to use the typical interface elements, and allowing for the use of structured, geometry-independent and relatively coarse meshes. The accuracy and capability of the proposed model are evaluated by verification and numerical examples covering water, DNAPL and $CO_2$ leakage through layers of different hydraulic properties.



## 1. Introduction

Leakage of a non-wetting phase through a porous medium domain constituting heterogeneous layers can have a significant consequence on the environment and life on earth. Leakage of contaminants, infiltration of dense oil and Leakage of $CO_2$ to the ground surface or layers containing ground water, among many others, are currently considered one of the main concerns of exploiting the earth space to cope with the current technological advancement.

Designing oil and gas fields, planning contaminant storages and selection of an appropriate geological formation for $CO_2$ sequestration require a good estimate of the amount of leakage that might take place in time. It is therefore vital to acquire computational tools capable of modeling this phenomenon. Modeling the leakage phenomenon accurately would not only give a good estimate of the amount of the leakage, but also an accurate approximation of the pore pressure distribution in the ground, and hence an accurate estimation of the mechanical behavior of the region surrounding such projects.

Computational modeling of multiphase flow in geological formations often requires modeling heterogeneous porous medium domains of regional scales with irregular and complicated geometry. Discretization of such a geometry is rather demanding. It requires finite element meshes (finite difference or finite volume grids), which are relatively fine and aligned along the boundaries between the layers. As the layers usually differ in porosity, permeability, and capillary entry pressure, fields generated by the fluid flow exhibit a jump at the boundary between them. This effect, in many cases, cannot be captured by standard numerical discretization schemes.

The physics of fluid leakage at boundaries between layers with different hydraulic properties has been intensively studied by several researchers, including Van Duijn et al. [1], Helmig and Huber [2], Van Duijn et al. [3], and Fučík and Mikyška [4]. The capillary pressure plays an important role in the amount of leakage between two layers. neighboring layers in a heterogeneous layered medium have different capillary pressure–saturation relationships. Fig. 1 shows typical Brooks and Corey capillary pressure–saturation relationships [5] for two layers having different permeability.

To illustrate the effect of capillary pressure on fluid flow in heterogeneous layered domain, a layered porous medium occupied by a wetting phase (water) that is being displaced by a non-wetting

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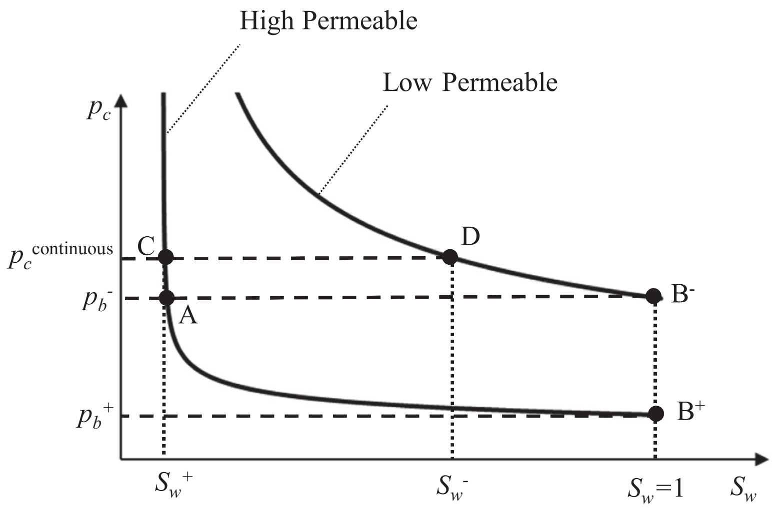


**Fig. 1.** Brooks and Corey capillary pressure–saturation relationships for two layers.

phase ($CO_2$, for example) is considered. In such a medium, according to Brooks and Corey capillary pressure–saturation relationships, the following conditions exist at the boundary between two layers:

- The non-wetting phase does not leak from a layer of high permeability to a layer of low permeability unless the capillary pressure of the first layer exceeds a threshold pressure, known as the entry pressure (also called bubbling pressure), of the second layer. $B^+$ in Fig. 1 indicates the entry pressure of the high permeability layer, and $B^-$ indicates that of the low permeability layer. This condition gives rise to mass accumulation of the non-wetting phase at the boundary between the two layers.
- Accumulation of the non-wetting phase continues to occur for all capillary pressures between point $B^+$ and point A in Fig. 1. In this region the capillary pressure at the boundary between the two layers exhibits a discontinuity.
- Above point A, the non-wetting phase starts to infiltrate into the second layer. In this region, the capillary pressure is continuous, and as a result, the saturation field exhibits a discontinuity. The capillary pressure crossing points C and D, in Fig. 1 is an example of this condition. It can be seen that these two points correspond to water saturations $S_w^+$ and $S_w^-$, respectively.
- If the non-wetting phase flow occurs from the low permeability layer to the high permeability layer, the saturation field also exhibits a jump, but in this case in the form of suction. Initially, at $S_w = 1$, the entry pressure of the low permeability layer ($B^-$) is readily higher than that of the high permeability layer ($B^+$). Due to this, upon the arrival of the non-wetting phase to the boundary between the two layers, leakage (suction) immediately occurs, maintaining $S_w = 1$ in the low permeability layer and decreasing in the high permeability layer.

Note that the van Genuchten capillary pressure–saturation relationship [6] exhibits continuous capillary pressure at all times. However, as for Brooks and Corey, the van Genuchten relationship exhibits the saturation discontinuity at the boundary between layers. In this paper, we utilize the Brooks and Corey relationship, though extension to van Genuchten is straightforward.

The presence of these complicated physical conditions at the boundary between heterogeneous layers exerts sever difficulties on the numerical solution procedure. The standard Galerkin finite element method (SG), for instance, is not able to simulate this problem accurately, even if a fine mesh is utilized. Helmig and Huber [2] intensively studied this problem and found that using SG to solve the infiltration of a Dense Non-Aqueous Phase Liquid (DNAPL) into a heterogeneous layered domain produces erroneous results. It fails to capture the discontinuity in the saturation field at the boundary between two layers, giving an incorrect impression of the amount of leakage.

Therefore, in order to solve such a problem, the numerical scheme must be able to capture the discontinuity in the capillary pressure and saturation fields. In literature, several solution techniques with different discretization complexities have been proposed. Friis and Evje [7], Brenner et al. [8], Cances [9] and Szymkiewicz et al. [10] used the finite volume method for this purpose. Helmig and Huber [2] used the subdomain collocation finite volume method (Box Method) to solve the problem. This method comprises coupling between the finite element method and the finite volume method. Fučík and Mikyška [4] utilized a mixed hybrid finite element-discontinuous Galerkin discretization procedure (MHFE-DG).

Here, we solve this problem using a mixed finite element discretization scheme. This scheme differs from the well-known mixed FEM, such that in the mixed FEM, different state variables are utilized but a single discretization technique is adopted. However, in the mixed discretization scheme, we utilize different state variables and adopt different discretization techniques, depending on the nature of the state variable and the associated balance equations. We use the partition of unity finite element method (PUM) [11] to discretize the discontinuity in the non-wetting phase saturation field, and the standard Galerkin method (SG) to discretize the continuous water (wetting phase) pressure. We adopt the partition of unity property within the framework of the extended finite element method (XFEM), which entails decomposing the saturation field into a continuous part and a discontinuous part, where the latter is enhanced by use of a function which closely describes the nature of the jump in the field (the Heaviside function in case of strong discontinuity, for instance). The main advantages of this method is two-folds. First, it captures the discontinuity accurately. Second, the discontinuity at the boundary between layers can be modeled regardless of the finite element mesh. Therefore, the mesh is not restricted to be aligned with the discontinuity, enabling the use of structured, geometry-independent and relatively coarse meshes.

This paper is organized as follows. In Section 2, governing equations based on a wetting pressure – non-wetting saturation formulation are derived. In Section 3, a detailed finite element