Advances in Water Resources 70 (2014) 65–76

Contents lists available at ScienceDirect

Advances in Water Resources

journal homepage: www.elsevier.com/locate/advwatres

Radial gravity currents in vertically graded porous media: Theory and experiments for Newtonian and power-law fluids

Vittorio Di Federico [a], Sandro Longo [b,*], Luca Chiapponi [b], Renata Archetti [a], Valentina Ciriello [a]

[a] Dipartimento di Ingegneria Civile, Chimica, Ambientale e dei Materiali (DICAM), Università di Bologna, Viale Risorgimento, 2, 40136 Bologna, Italy

[b] Dipartimento di Ingegneria Civile, Ambiente Territorio e Architettura (DICATeA), Università di Parma, Parco Area delle Scienze, 181/A, 43124 Parma, Italy

ARTICLE INFO

Article history:
Received 22 December 2013
Received in revised form 17 April 2014
Accepted 21 April 2014
Available online 30 April 2014

Keywords:
Porous
Gravity current
Similarity solution
Variable permeability
Non-Newtonian
Laboratory experiments

ABSTRACT

This study theoretically and experimentally explores the behaviour of axisymmetric gravity currents of Newtonian and power-law fluids in inhomogeneous porous media. Systematic heterogeneity along the vertical is represented by a power-law permeability variation governed by the parameter ω, mimicking trends in natural media. A self-similar solution describing (i) the rate of propagation and (ii) the profile of the current is derived by considering a current of volume proportional to time raised to a non-negative power α. Four critical values of α are determined: the first two affect the time dependency of the radius, height and average gradient of the current on flow behaviour index n and ω; the third one dictates if the current accelerates or decelerates; the fourth one governs the asymptotic validity of the thin current approximation. Experimental validation is performed using shear-thinning suspensions and Newtonian mixtures in constant- and variable-flux regimes. A stratified porous medium composed of four uniform strata of glass beads with different diameters is used for this purpose. The experimental results for the radius and profile of the current agree well with the self-similar solution except at the beginning of the process, due to the limitations of the 1-D assumption and to boundary effects near the injection zone. An uncertainty analysis indicates that the rheological fluid behaviour and the variation in permeability significantly affect the propagation of the current.

1. Introduction

Extensive research has been conducted on gravity-driven motion through porous media. These studies have been motivated by several geophysical and industrial applications, including enhanced oil recovery, contaminant migration, seawater intrusion, and carbon dioxide sequestration in geological formations [1–4]. The behaviour of porous gravity currents is generally analyzed by considering the release of a time-variable volume of an intruding fluid in an infinite domain under the thin current assumption. Solutions in self-similar form were derived by Huppert and Woods [5] for plane geometry and by Lyle et al. [6] for axisymmetric geometry. Di Federico et al. [7,8] recently extended these studies to non-Newtonian flow, to handle the complex rheological nature of many fluids involved in relevant applications. These include injection of displacing suspensions or muds in enhanced oil recovery and well drilling [9,10], crude oil flow in reservoirs [11], soil remediation via nanoparticles advected by fluid carriers [12], subsurface contamination by polymeric pollutants (e.g. [13] and references therein), soil grouting [14], flow of blood in biological porous media [15], blood filtration through reticulated foams [16]. Diverse rheological models are available in the literature for the description of non-Newtonian behaviour [17]; among these, the simplest is represented by the two-parameter power-law model. This formulation usually provides an accurate approximation in the intermediate shear rate range, as demonstrated in e.g. Longo et al. [18], where a power-law model satisfactorily fitted rheometric measurements of shear-thinning fluids in the interval 0.1–5 s^{-1}.

In addition to rheological fluid behaviour, the propagation of gravity driven flow in natural porous formations is strongly affected by heterogeneity [19]. Vertical permeability and porosity gradients have been shown to condition front propagation in plane fluid drainage from an edge [20]. In a two-layered porous medium and above a critical influx, the intruding fluid overrides the low-permeability lower layer, enhancing mixing [4]. Investigation into the combined effects of fluid rheology and spatial permeability variations is crucial in several applications in natural porous media (e.g. [21]). The present study focuses on systematic permeability variations of the kind extensively adopted in the porous media literature [2,20,22–24]; these closed-form expressions

* Corresponding author. Tel.: +39 0521 90 5157; fax: +39 0521 90 5924.
E-mail address: sandro.longo@unipr.it (S. Longo).

http://dx.doi.org/10.1016/j.advwatres.2014.04.015
0309-1708/

approximately mimic trends occurring in natural media. A coupled theoretical and experimental approach is used here to analyse the influence of vertical permeability gradients perpendicular to the flow direction on axisymmetric non-Newtonian power-law gravity currents with time-variable inflow. First, the problem is formulated in dimensionless form (Section 2.1) and a similarity solution that generalises the results of Di Federico et al. [8] is derived (Section 2.2). Second, the dependency of the radius and height of the current on problem parameters is discussed (Section 3). The theoretical solution was tested against data from laboratory experiments conducted with shear-thinning suspensions and Newtonian mixtures in constant- and variable-flux regimes; the experimental setup is described in Section 4.1, while the results of the experiments are presented and compared with the theory in Section 4.2. The goodness of the approximation provided by the proposed formulation is examined via an ad hoc uncertainty analysis (Section 4.3). A set of conclusions closes the paper (Section 5).

2. Theoretical model

2.1. Formulation

Consider the setting depicted in Fig. 1, in which r and z represent radial and vertical coordinates respectively. An axisymmetric gravity current of a non-Newtonian fluid of uniform density ρ, with rheology described by a power-law model $\tau = m|\dot{\gamma}|^{n-1}\dot{\gamma}$, with τ and $\dot{\gamma}$ the shear stress and rate, m the consistency index, and n the flow behaviour index, is released at the origin and intrudes into an infinite porous domain of depth h_0 saturated with another fluid of uniform density $\rho - \Delta\rho$. The intruding current, described by its height $h(r,t)$ in the sharp interface approximation, extends above a horizontal impermeable bed to a coordinate denoted by $r_N(t)$. We consider the case of a isotropic heterogeneous domain in which the medium permeability k (dimensions $[\mathrm{L}^2]$) is constant in the horizontal direction but has a vertical gradient described by [2]

$$k(z) = k_0(z/r^*)^{\omega-1}, \tag{1}$$

where k_0 is a characteristic permeability, r^* is a length scale, and ω is a constant. Values of $\omega < 1,\ \omega = 1$ and $\omega > 1$ represent negative, null, and positive gradients with elevation, respectively. A lower bound is set to the value of ω for assigned n, i.e. $\omega > \omega_0 = (n-1)/(n+1)$. This ensures the validity of the self-similar solutions derived in the sequel; physically, it is equivalent to limit the permeability decrease with elevation. For a Newtonian fluid ($n = 1$), $\omega_0 = 0$, as earlier noted by Ciriello et al. [2] and Mathunjwa and Hogg [25]. Note that the permeability tends to decrease with depth in natural porous and fractured media [22,23], rendering the case $\omega \geqslant 1$ decidedly more common than $\omega < 1$. We also assume that capillary effects are negligible and that the thin current approximation holds, which allows us to disregard motion in the ambient fluid and vertical velocities in the intruding fluid. Under these assumptions, the pressure distribution in the intruding current is hydrostatic and given, for $0 \leqslant z \leqslant h$, by $p(r,z,t) = p_1 + \Delta\rho g h(r,t) - \rho g z$, where $p_1 = p_0 + (\rho - \Delta\rho) g h_0$ is a constant and p_0 is the constant pressure at $z = h_0$.

The equation of motion of a non-Newtonian power-law fluid in a porous medium is given by [26,27]

$$\nabla p - \rho \mathbf{g} = -\frac{\mu_{eff}}{k} |\,\boldsymbol{u}|^{n-1}\boldsymbol{u} \tag{2}$$

where p is the pressure, $\boldsymbol{u}$ the Darcy velocity, $\mathbf{g}$ the acceleration due to gravity, and μ_{eff} the effective viscosity (dimensions $[\mathrm{M\,L}^{-n}\,\mathrm{T}^{n-2}]$). The inverse of the proportionality factor that appears in (2) is termed 'mobility' and is expressed as [8]

$$\frac{k}{\mu_{eff}} = \frac{1}{2C_t}\frac{1}{m}\left(\frac{n\phi}{3n+1}\right)^n \left(\frac{50k}{3\phi}\right)^{(n+1)/2}, \tag{3}$$

where ϕ and C_t (> 1) represent medium porosity and tortuosity, respectively. The latter factor empirically accounts for the complex nature of non-Newtonian fluid flow in porous media. As such, it has been expressed in several ways in the literature in the form $C_t = C_t(n)$ [28], with the various formulations differing significantly. The expression proposed by Pascal [29], i.e., $C_t = (25/12)^{(n+1)/2}$ simplifies the mobility expression to $k/\mu_{eff} = (1/(2m))(n\phi/(3n+1))^n(8k/\phi)^{(n+1)/2}$ and, for a Newtonian fluid ($n = 1$), allows (2), combined with (3), to reduce to Darcy's law, $\nabla p - \rho\mathbf{g} = -(\mu/k)\mathbf{u}$, where μ is the dynamic viscosity. Pascal's formulation for the tortuosity is adopted in the interpretation of the experimental results. The hydrostatic assumption allows the pressure gradient to be expressed as a function of the unknown free surface as $\partial p/\partial r = \Delta\rho g(\partial h/\partial r)$, which, together with (1) and (2), yields the following expression for Darcy velocity in the r direction for purely horizontal flow:

$$u(r,z,t) = -(\Lambda\Delta\rho g)^{1/n} k_0^{(n+1)/(2n)} \left(\frac{z}{r^*}\right)^{\frac{(\omega-1)(n+1)}{2n}} \left|\frac{\partial h}{\partial r}\right|^{1/n-1} \frac{\partial h}{\partial r}, \tag{4}$$

$$\Lambda = \Lambda(\phi, m, n) = \frac{1}{2C_t}\left(\frac{50}{3}\right)^{(n+1)/2}\left(\frac{n}{3n+1}\right)^n \frac{\phi^{(n-1)/2}}{m}, \tag{5}$$

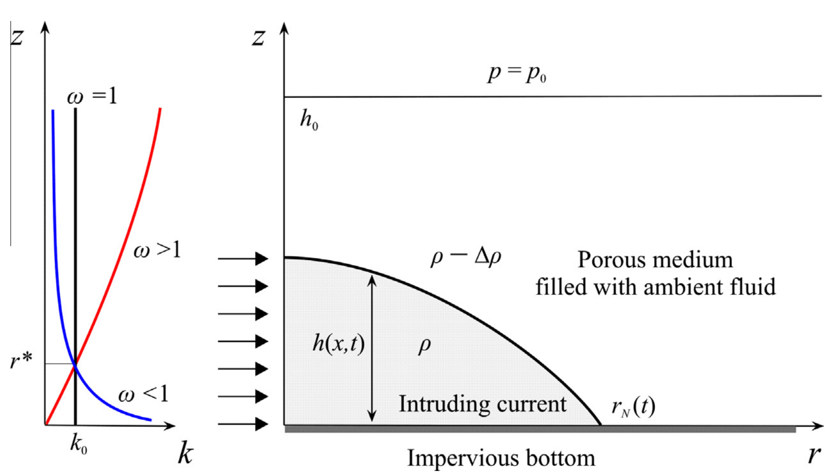

Fig. 1. Sketch of an axisymmetric gravity current intruding into a saturated porous medium of thickness h_0. The left panel illustrates vertically increasing ($\omega > 1$), decreasing ($\omega < 1$) and homogeneous ($\omega = 1$) permeabilities.