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A semi-analytical method for predicting the outflow hydrograph due to dam-break in natural valleys

Francesca Aureli, Andrea Maranzoni *, Paolo Mignosa

University of Parma, DICATeA, Parco Area delle Scienze, 181/A, 43124 Parma, Italy

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ABSTRACT

The paper presents a semi-analytical method for predicting the flow rate hydrograph due to a hypothetical sudden and total dam failure in a natural valley. The method generalizes the approach proposed by Hunt for the dam-break problem in a rectangular frictionless sloping channel to a valley with a cross-section area expressed by a power-law function of water depth, in order to take into account the most common shapes of natural valleys. The parameters of the deriving model can be set by exploiting data usually available concerning the dam section geometry and the reservoir storage-depth curve. The application of the technique to three different reservoirs is discussed. The results show that the flow rate hydrographs obtained at the dam site agree with the ones calculated by means of a finite volume numerical code based on two-dimensional shallow water equations. The method requires moderate computational and data collecting effort, so it can be regarded as a useful alternative to other procedures commonly adopted in the practice.

1. Introduction

Dams and their reservoirs provide important benefits to human communities, including hydroelectric production and water storage for consumption, irrigation, downstream flood control and navigation, and recreation. Along with these benefits, however, dams pose serious flooding risks for downstream river reaches, agricultural lands, historical sites, industrial and urban areas because they create large water volumes, often located at a great height, that are rapidly released in case of a collapse, with potentially catastrophic consequences.

The estimation of the flood wave due to the breaking of a dam is usually a necessary requirement to predict the flood arrival time, the maximum water depth and velocity, and the floodable areas. The availability of this information actually allows flood hazard assessment and the development of civil protection plans for the arrangement of both corrective and preventative measures to reduce flood damage (EU Flood Directive 2007/60, [1]).

In real field applications characterized by irregular topography, dam-break waves must be analyzed by means of suitable numerical models. The few exact solutions available in the literature for the dam-break problem actually deal with schematic geometries, and, although theoretically valuable and useful for validating numerical models, are almost useless for practical purposes. The earliest and best known analytical solution was obtained by Ritter [2], who assumed a total and instantaneous dam collapse on a rectangular and horizontal dry channel in the absence of friction. Stoker [3] extended the Ritter solution assuming a non-zero downstream water depth. Anyhow, the hypothesis of horizontal channel implies an infinite impounded water volume and a constant discharge at the dam section. Later, in two different works Dressler investigated the effects of friction [4] and bed slope [5]. In the former work he adopted a perturbation procedure to modify the Ritter solution in the presence of friction, whilst in the latter he derived an integral solution for the dam-break problem on smooth sloping bed. Whitham [6] too studied the effects of hydraulic resistance. Su and Barnes [7] generalized the Ritter solution to the case of a channel cross-section expressed as a power-law function of water depth. However, once again the channel is assumed to be horizontal and friction neglected. For its simplicity and intrinsic descriptive strength, the idea of adopting a power-law function to describe channel cross-section geometrical characteristics was considered and implemented by many authors in the following years [9–15]. More recently new analytical solutions based upon the method of characteristics were developed by Chanson [8].

In order to assess the outflow discharge hydrograph caused by a dam-break, different approaches could be adopted, thereby entailing extremely different efforts both for what concerns the collection of the input data required and from a computational point of view. As an example, solving the equations governing rapidly varying free surface flow would require the availability of a validated computational tool, a hopefully accurate digital elevation

* Corresponding author. Tel.: +39 0521 905949; fax: +39 0521 905924.
E-mail addresses: francesca.aureli@unipr.it (F. Aureli), andrea.maranzoni@unipr.it (A. Maranzoni).

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model of the area under investigation (including the reservoir), and usually a non-negligible computational time. On the other hand simplified approaches proposed in the literature with this aim (NWS, [16], CASTOR, [17], [18], just to mention a few) are able to produce an expeditious, almost inexpensive, and reasonably accurate peak flow evaluation, sometimes under significant constraints due to the model limitations. Whenever a large number of scenarios needs to be considered to perform a comprehensive dam-break inundation analysis and downstream hazard classification (as imposed by current technical regulations in many countries), the adoption of a simplified approach would surely be advisable. Preferential requisites of a simplified procedure are certainly the need for basic computational tools and easily available input data, in addition to its grounding on a strong theoretical basis and its validation on the basis of the comparison with more accurate numerical and/or physical models.

Focusing their research on the characterization of the hydrograph at the breach section following a partial dam-break, Pilotti et al. [15] presented a simplified approach based on a suitable adimensionalization of numerical results obtained by means of a 2D shallow water model. More recently, Pilotti et al. [31] proposed an alternative procedure limited to the case of horizontal channel with rectangular cross-section.

A semi-analytical methodology is here presented with the aim of providing a realistic evaluation of the dam-break wave caused at the dam site by a sudden and total dam failure. The approach originally proposed by Hunt [19] for a rectangular frictionless sloping channel is generalized to derive a dimensionless solution for the dam-break wave in a valley characterized by a cross-section expressed as a power-law function of water depth. In this way the methodology aims to take into account the most common shapes of natural valleys, together with the reservoir storage-depth curve and the inertial effects, while requiring little geometrical information (usually available) and entailing simple computations. The discharge hydrograph obtained at the dam site can then be adopted as upstream boundary condition for the subsequent 1D or 2D modeling of the dam-break wave propagation in the downstream valley. This is clearly useful when a numerical computation of the dam-break including the reservoir is not possible. Actually, very often for existing reservoirs the volume-stage curve is the only information available and a detailed topographic survey of the bathymetry is missing.

The proposed procedure is validated on the basis of three significant test cases, among which the historical Malpasset dam-break event occurred in 1959. The outflow discharge hydrographs predicted by the method are compared with both the ones evaluated through a finite volume numerical scheme based on 2D shallow water equations and the ones calculated by adopting the level-reservoir approximation (LRA) coupled with the critical state assumption at the dam section.

2. Theoretical basis of the method

It is well known that the de Saint–Venant equations, widely used for modeling river flows, can be written in divergent vector form (e.g., [20,21]) as:

$$\frac{\partial \mathbf{U}}{\partial t}+\frac{\partial \mathbf{F}}{\partial x}=\mathbf{S},\quad \mathbf{U}=\begin{pmatrix}A\\Q\end{pmatrix},\quad \mathbf{F}=\begin{pmatrix}Q\\ \frac{Q^2}{A}+gI_1\end{pmatrix},\quad \mathbf{S}=\begin{pmatrix}0\\ gA(S_0-S_f)+gI_2\end{pmatrix},\tag{1}$$

where

$$I_1=\int_0^h (h-y)\,b(x,y)\,dy \quad\text{and}\quad I_2=\int_0^h (h-y)\,\frac{\partial b}{\partial x}\,dy \tag{2}$$

are the first order moment of the wetted area with respect to the free surface and the non-prismaticity term. In (1) and (2) x, t, g, h, S_0, S_f e $b(x,y)$ represent: the distance along the flow direction, the time variable, the acceleration due to gravity, the water depth, the bed slope, the friction slope and the cross-section width at depth y, respectively.

Let us consider the rapidly varying free surface flow consequent to the instantaneous breaking of a dam (initially retaining a maximum water depth h_0) in a prismatic ($I_2=0$), frictionless ($S_f=0$), sloping channel (see Fig. 1). With the speed of small amplitude waves in still water, c, and the mean flow velocity, u, as dependent variables, it is possible to obtain the characteristic form of the governing equations:

$$\begin{cases}\frac{dx_\pm}{dt}=u\pm c\\ \frac{d}{dt}(u(x_\pm,t)\pm\omega(x_\pm,t))=gS_0\end{cases}\tag{3}$$

in which

$$c=\sqrt{g\frac{A(h)}{b(h)}}\quad\text{and}\quad \omega=\int_0^h\sqrt{g\frac{b(y)}{A(y)}}dy \tag{4}$$

are the wave celerity and the Escoffier stage variable [22,23], respectively.

By assuming the following power-type expression for the relationship between wetted area and water depth:

$$A=\delta h^\lambda, \tag{5}$$

the celerity and the Escoffier stage variable become:

$$c=\frac{\sqrt{gh}}{\sqrt{\lambda}},\quad \omega=\int_0^h\sqrt{g\lambda}\frac{1}{\sqrt{y}}\,dy=2\sqrt{\lambda}\sqrt{gh}=2\lambda c. \tag{6}$$

In Eq. (5) the exponent λ acts as a shape parameter allowing to describe cross-section shapes typical of the most common geometries of natural valleys; in particular $\lambda=1$, $\lambda=1.5$, and $\lambda=2$ refer to a rectangular, parabolic and triangular cross-section, respectively. Moreover, the storage volume is given by:

$$V(h)=\int_0^h\frac{\delta y^\lambda}{S_0}dy=\left[\frac{\delta}{(\lambda+1)\cdot S_0}\right]h^{\lambda+1}=\eta h^{\lambda+1}. \tag{7}$$

In case of horizontal channel ($S_0=0$), the closed-form solution of the frictionless dam-break problem is provided by the following relations obtained by Su and Barnes [7]:

$$\frac{u}{\sqrt{gh_0}}=2\beta+\alpha\frac{x}{t\sqrt{gh_0}},\qquad \frac{c}{\sqrt{gh_0}}=\frac{1}{\sqrt{\lambda}}\left(\alpha-\beta\frac{x}{t\sqrt{gh_0}}\right), \tag{8}$$

with

$$\alpha=\frac{2\lambda}{2\lambda+1}\quad\text{and}\quad \beta=\frac{\sqrt{\lambda}}{2\lambda+1}, \tag{9}$$

which provide a complete description, in space and time, of the u- and c-profiles. For a rectangular cross-section ($\lambda=1$), expressions (8) reduce to the well-known Ritter solution [2].

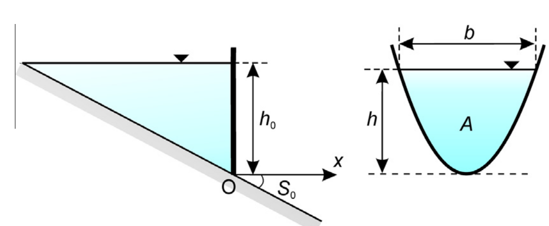

Fig. 1. Definition sketch for the dam-break problem.