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A novel semi-analytical approach for non-uniform vegetated flows

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ABSTRACT

In this paper a semi-analytical approach is proposed to understand the mechanism by which a non-uniform vegetated flow passes over a finite thick soil layer covered with grass. The flow region is divided into three layers: a homogenous water layer, a mixed water-grass layer, and a finite thick soil layer (hereafter referred to as the water layer, the grass layer, and the soil layer). The flow of the water layer is governed by the Navier-Stokes equations. Both the grass and soil layers are regarded as porous media and the Biot's theory of poroelasticity is applied to the porous medium flow. The semi-closed solutions are then obtained by the Runge-Kutta method. The drag force induced by the flow through the grass layer and the flow profiles of three patterns: submerged grass, emergent grass and mixed type are also discussed.

1. Introduction

Vegetated flow in open channels or wetlands has received considerable attention in the literature. When surface water flow passes over an area of vegetation, the flow conditions were mostly divided into two types– submerged and emergent. Huai et al. [9] employed a three-layer model to investigate the vertical velocity distribution of open channel flow with submerged rigid vegetation. Zhao et al. [19] considered a uniform laminar flow through submerged and floating plants in wetlands by an analytical approach. Kubrak et al. [12] developed a one-dimensional steady model to study the vertical velocity profiles of channel flow through submerged or emergent vegetation. In their study, the resistance to the water flow caused by vegetation and the roughness of the channel bottom and vegetation were discussed. Wu et al. [16] studied the variation of roughness coefficients for turbulent flow through emergent or submerged vegetation by using a horsehair mattress to experimentally simulate the vegetation on watercourses. In their study, a simplified model based on force equilibrium was developed to evaluate the drag coefficient of the vegetative element and the Manning equation was then employed to convert the drag coefficient into the roughness coefficient.

On the other hand, Wu et al. [17] and Wu and He [18] presented the concept of drag force and flexural rigidity to investigate the effect of the roughness and density of flexible vegetation and rigid vegetation on the turbulent channel flow, respectively. They proposed a hydraulic model to estimate the flow rate of a channel, and then verified the model by the experimental data and field data of two natural rivers. Finally, they applied the model to the study of sediment transport in a vegetated channel. Hsieh and Shiu [6] employed the Biot's theory of poroelasticity [2–4] to investigate the surface and subsurface flow by regarding the vegetation zone as a porous medium and derived the vertical velocity distribution. In their study, the effect of rainfall was not taken into account. Recently, Hsieh and Yang [7] and Hsieh et al. [8] developed a series of integrated solutions to the overland and subsurface flow along a sloping soil layer to discuss the effects of rainfall on the water flow for various soil layer thicknesses. However, the effects of vegetation were not considered in these two studies.

Based on the above comments, this study is proposed to investigate the mechanism of non-uniform vegetated flow under a uniform rainfall. In addition to the two types of flow conditions, emergent and submerged, a third one, mixed type: partially submerged and partially emergent, is also discussed. These conditions usually occur in surface water flow down a slope under a rainfall. This work is only restricted to laminar flow, but it can be regarded as the pioneer research for the future turbulent flow study.

2. Problem Formulation

Fig. 1 depicts a two-dimensional surface flow passing over a vegetated area under a uniform rainfall. In the figure, three layers are evident: a water layer, a grass layer and a soil layer. The fluid is assumed viscous, incompressible, and homogeneous. The grass and soil layers are assumed homogeneous, and the motion of the pore

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Nomenclature

$C_1 \sim C_6$	undetermined coefficients	n_2,n_3	porosity of vegetation layer and soil layer, respectively
D_f	drag force per unit length	p_1	hydrostatic pressure in water layer
D_{fM}	maximum drag force per unit length	p_2,p_3	pore water pressure in vegetation and soil layers, respectively
d_c	average diameter of the stems	s	slope of the ground
G,λ	Lame constants of elasticity	$\bar{u}$	mean velocity of flow through the vegetation layer
g	gravitational acceleration	u_1	velocity in water layer in the x direction
H	thickness of the soil layer.	u_2	velocity in vegetation layer in the x direction
h_1	depth of homogeneous water layer	u_3	velocity in soil layer in the x direction
h_2	height of vegetation	u_{Hsieh}	maximum velocity of the surface flow over a bare slope
i	rainfall intensity	μ	dynamic viscosity of water
j	subscript denoting the jth layer	θ	inclined angle of the ground
k_{p2},k_{p3}	specific permeability of vegetation layer and soil layer, respectively	ρ	density of water

water is regarded as the porous medium flow. All the physical variables and parameters are averaged over a representative elementary volume (REV), and thus the flow inside the porous medium can be described by means of partial differential equations (referring to Bear [1]). Moreover, the surface and subsurface flow is assumed to reach a steady state. The vertical velocity of the flow can be ignored when the soil layer is saturated. The flow length is considerably greater than the flow depth so that the streamwise derivatives are smaller than the transverse derivatives, and thus they can be ignored.

2.1. Governing Equations

Based on the above assumptions, if the water layer is shallow, most of the impact momentum will be transferred to the underlying soil (Van der Molen et al. [15]). If the transfer of momentum from raindrops to the water layer is ignored, the Navier-Stokes equations can be simplified to:

$$x - \text{dir.}: \mu\frac{\partial^2 u_1}{\partial y^2} - \frac{\partial p_1}{\partial x} + \rho g sin\theta = 0, -\infty < x < \infty, h_2 \leqslant y \leqslant h \quad (1)$$

$$y - \text{dir.}: -\frac{\partial p_1}{\partial y} - \rho g cos\theta = 0, -\infty < x < \infty, h_2 \leqslant y \leqslant h \quad (2)$$

Where u_1 is the fluid velocity in the $x -$ direction and p_1 is the fluid pressure of the water layer (i.e. layer 1); ρ and μ are the fluid density and dynamic viscosity respectively; g is the gravitational acceleration; θ is the slope angle; $h_1(x)$ is the thickness of the water layer; h_2 is the height of the grass; $h(x)$ is the water depth; and x and y are the coordinates as shown in Fig. 1.

According to the work of Hsieh and Bolton [5] who simplified the Biot's theory of poroelasticity [2,3] to simulate a low Reynolds number flow over vegetated sloping ground, the momentum

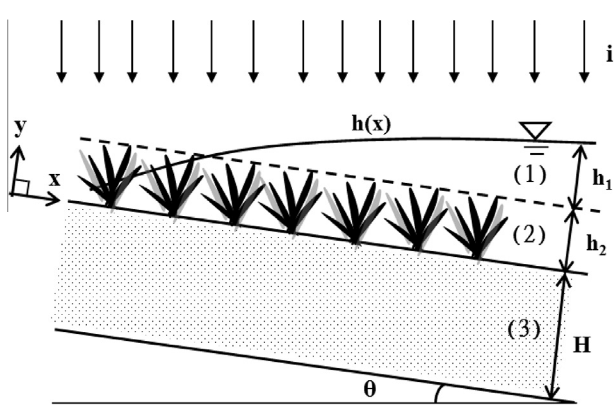

Fig. 1. Schematic diagram of the study.

equations of the pore water within the REV inside porous media may be represented as

$$x - \text{dir}: n_j\mu\frac{\partial^2 u_j}{\partial y^2} - \frac{\mu n_j^2}{k_{p_j}}u_j - n_j\frac{\partial p_j}{\partial x} + n_j\rho g\sin\theta = 0, \quad (3)$$

$$y - \text{dir}: -n_j\frac{\partial p_j}{\partial y} - n_j\rho g\cos\theta = 0, \quad (4)$$

Where u_j is the velocity component of the pore water in the x-direction of layer j; p_j is the pore water pressure inside the porous medium of layer j; n_j is the porosity of layer j; k_{pj} is the specific or intrinsic permeability coefficient of layer j; the subscript $j = 2$ for the grass layer ($0 \leqslant y \leqslant h_2$); the subscript $j = 3$ for the soil layer ($-H \leqslant y \leqslant 0$); and H is the thickness of the soil layer.

2.2. Boundary Conditions

Based on the assumptions of no wind force, no surface water waves, steady flow, and using gauge pressure, six boundary conditions are examined as follows.

(1) at the free surface ($y = h(x)$):
 (i) continuity of fluid stress in the normal (y) direction

$$p_1 = 0 \quad (5)$$

 (ii) continuity of fluid stress in the tangential (x) direction

$$\mu\frac{\partial u_1}{\partial y} = 0 \quad (6)$$

(2) at the interface between the water layer and the grass layer ($y = h_2$):
 (i) continuity of velocity in the tangential (x) direction

$$u_1 = n\ u_2 \quad (7)$$

 (ii) continuity of fluid stress in the tangential (x) direction

$$\mu\frac{\partial u_1}{\partial y} = \mu\frac{\partial u_2}{\partial y} \quad (8)$$

 (iii) continuity of fluid stress in the normal (y) direction

$$p_1 = p_2 \quad (9)$$

(3) at the interface between the grass layer and the soil layer ($y = 0$):
 (i) continuity of velocity in the tangential (x) direction

$$n_2 u_2 = n_3\ u_3 \quad (10)$$

 (ii) continuity of fluid stress in the tangential (x) direction

$$\mu\frac{\partial u_2}{\partial y} = \mu\frac{\partial u_3}{\partial y} \quad (11)$$