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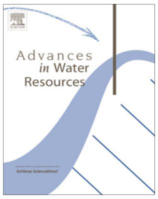

Binary upscaling on complex heterogeneities: The role of geometry and connectivity

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ABSTRACT

The equivalent conductivity (K_{eq}) of a binary medium is known to vary with the proportion of the two phases, but it also depends on the geometry and topology of the inclusions. In this paper, we analyze the role of connectivity and shape of the connected components through a correlation study between K_{eq} and two topological and geometrical indicators: the Euler number and the Solidity indicator. We show that a local measure such as the Euler number is weakly correlated to K_{eq} and therefore it is not suitable to quantify the influence of connectivity on the global flux; on the contrary the Solidity indicator, related to the convex hull of the connected components, presents a direct correlation with K_{eq}. This result suggests that, in order to estimate K_{eq} properly, one may consider the convex hull of each connected component as the area of influence of its spatial distribution on flow and make a correction of the proportion of the hydrofacies according to that. As a direct application of these principles, we propose a new method for the estimation of K_{eq} using simple image analysis operations. In particular, we introduce a direct measure of the connected fraction and a non-parametric correction of the hydrofacies proportion to compensate for the influence of the connected components shape on flow. This model, tested on a large ensemble of isotropic media, provides a good K_{eq} approximation even on complex heterogeneities without the need for calibration.

1. Introduction

The adoption of Darcy's law for the description of a macroscopic flow through porous media is commonly accepted, but the problem of finding a representative hydraulic conductivity arises in the case of a heterogeneous medium. This type of problem and the proposed solutions are concerning all the physical processes which follow the same type of laws, e.g. electricity or heat conductance. Regarding hydrogeology, the subject is of primary importance for hydrocarbon reservoir and hydrological basins flow modeling, since local parameterization needs to be conveniently upscaled to represent large-scale properties in the model. The upscaling can be done by substituting each volume of heterogeneous sediments with a homogeneous medium characterized by an equivalent hydraulic conductivity (K_{eq}) value in order to present the same mean flow response. In a complex aquifer, this value is a function of the small-scale hydraulic conductivities and the proportion of each hydrofacies, but also of their sub-scale geometry [1]. The arithmetic and the harmonic mean of the local conductivities are known to be the widest upper and lower K_{eq} bounds respectively [2]. These bounds are referred to as Wiener bounds and correspond to the cases in which the flow is parallel or perpendicular to a set of plain layers of different conductivities. In the case of a binary and statistically isotropic medium, the K_{eq} range can be reduced to the Hashin–Shtrikman bounds [3]. The Effective Medium Theory (*EMT*), based on Maxwell formula [4] on spherical inclusions, gives an exact analytical solution for the effective conductivity K_{eff} of a very dilute suspension of inclusions in a homogeneous matrix of conductivity K_0, in which the mean flow is uniform and governed by the matrix. This formula, then developed by Matheron [5], Dagan [6,7] and applied to bimodal formations by Rubin [8], has been recently generalized for three-dimensional heterogeneous medium of log-normal distribution [9], giving an accurate approximation for denser configurations of the inclusions [10], different radius [11], shape and distribution [12]. The main limit of this approach lies in some assumptions that are not satisfied when the integral scale of the inclusions material is large with respect to the size of the upscaled volume, the non-linear interactions between the inclusions become more and more important and the estimation less accurate.

Many other approaches have been developed for the study and estimation of K_{eq} including for example, the stochastic theory applied to random multi-Gaussian fields [13,14], power averaging equations [15], homogenization theory [16] and renormalization [17,18]. For an extensive overview about these methods see [19,20]. These techniques give accurate results for specific types

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of heterogeneities and do not provide a thorough topological and geometrical analysis. Recently, the topology and geometry of the sub-scale structure has been recognized to be of crucial importance for the upscaled conductivity [21–25]. Knudby et al. [26] developed a model for the estimation of K_{eq} in random binary fields containing multiple inclusions. Their formula, derived from the empirical observations of Bumgardner [27], is based on a static connectivity measure: the mean distance between the inclusions along the mean flow direction. This approach leads to good estimations for fields presenting isolated inclusions however it is not flexible enough to deal with more complex heterogeneities, for instance channeled textures or non-convex inclusions penetrating each other. Finally, Herrmann and Bernabé [28,29] proposed a site percolation model for binary fields that accounts for the connectivity of the more conductive hydrofacies as a function of its proportion. This parametric approach can be applied on known stochastic fields, otherwise it has to be calibrated through physical or numerical experiments.

The aim of this paper is to analyze how the geometry and connectivity of heterogeneities influence the equivalent hydraulic conductivity (K_{eq}) of isotropic binary media. For this purpose, we perform an analysis of the correlation between K_{eq} and two topological and geometrical indicators: the Euler number and the Solidity indicator. This is done on a group of 2D binary fields which present a large variability of these indicators. Furthermore, we propose a new method based on image analysis to estimate K_{eq} for isotropic binary fields. The proposed algorithm is tested mainly on 2D fields, but an early 3D implementation of the algorithm is also presented. The proposed approach is essentially empirical. It finds its roots in the correlation studies and the analysis of the flow fields from numerical simulations. This method allows to rapidly estimate the equivalent, or "block", conductivity on any given isotropic binary field, using no information about the underling statistical model. The analysis is conducted on 2D realizations of stochastic fields and 3D natural heterogeneities without assuming stationarity, mean uniform flow or restrictions on the integral scale of the conductivity field.

The paper is organized as follows: in Section 2 the techniques used for the generation of the binary conductivity fields, the computation of the spatial indicators and the reference K_{eq} values are described. In Section 3, the results obtained from the correlation studies are presented and discussed. In Section 4, the new formula for the estimation of K_{eq} is presented together with its application on complex heterogeneities. Section 5 is devoted to conclusions.

2. Materials and methods

In this section, we describe the different steps required to make a quantitative analysis of the influence of geometry and topology on the equivalent conductivity and develop a K_{eq} estimation model. The preliminary part consists in the generation of several groups of two-dimensional binary fields, presenting isotropic textures and varying proportion, shape and connectivity values for each hydrofacies. This gives us a wide basis for our correlation study. Second, the equivalent conductivity is computed performing flow simulations on the generated fields and it is used as reference. Third, the Euler number of the more conductive hydrofacies is calculated for each field and adopted as connectivity indicator. Fourth, the average Solidity indicator is computed and used as a geometrical indicator. Finally, a correlation study between K_{eq} and these indicators is performed and an experimental algorithm to estimate K_{eq} based on image analysis is proposed as an application of the information achieved through the correlation study.

2.1. Generation of binary fields

The binary fields used in this study are composed of a highly permeable material (represented by the symbol *HP* in the rest of the text, the value 1 in the binary fields and the white color in the figures) with a hydraulic conductivity value $k_h = 5 \times 10^{-2}$ (m/s) and a less permeable one (*LP*, value 0, black color) with $k_l = 5 \times 10^{-6}$ (m/s). In the first part of the study the aim is to vary one spatial indicator at a time (e.g. varying the Euler number and keeping constant the Solidity indicator and the hydrofacies proportion) in order to investigate its correlation with K_{eq}. This is achieved by adding randomly placed non-touching inclusions of one hydrofacies on a clean background until the desired proportion p of the inclusions is reached. In this way, one can control the level of connectedness indirectly by varying the dimension, number and minimal distance between the connected components, while keeping a constant proportion p, or control the geometry choosing among any type of shape (Table 1, tests 1 and 2). In the second part, the aim is to test the new method of K_{eq} estimation as systematically as possible on fields presenting both simple and complex geometries. For this purpose, a group of 2D Bernoulli fields (Table 1, test 3) is obtained by imposing different threshold values on 400 arrays composed of uniformly distributed pseudo-random numbers. All the range of proportion $p \in [0, 1]$ is covered and the obtained fields are statistically isotropic.

Moreover, we generate 10,800 binary images presenting various types of texture with the following procedure (Table 1, test 4):

1. We start from a group of 20 realizations of a 2D multivariate Gaussian random model, simulated using a Gaussian variogram with a correlation length of 40 pixels.
2. Using the technique proposed by Zinn and Harvey [22], each realization is transformed to obtain two different fields: in the first one, continuous channels are formed by the minima of the 2D multivariate Gaussian random function and, in the second one, the same type of structures are formed by the maxima.
3. 18 combinations of coupled threshold values are imposed on each field to obtain different types of binary, generally well connected, distributions. In order to maximize the geometrical and topological variability among the binary images, the threshold values are computed as $T = (D + S)/2$, where D is a vector of values taken at regular intervals in the range of the generated variable and S is a vector of equally distant quantiles of its empirical probability distribution.
4. Finally, each of these images is edited using the matlab function *Randblock* (Copyright 2009 Jos vander Geest), which divides the image in squares of the same size and randomly mixes them. We use this tool to obtain several fields with the same material proportions but different geometries and connectivities. This operation is repeated 10 times, progressively reducing the square size to obtain finer textured mosaics.

This ensemble of techniques allows to cover the space of the possible (p, K_{eq}) solutions widely (see Section 4.1). Even if the starting images are isotropic, using Randblock may cause the formation of anisotropic media. For this reason, the fields presenting a ratio between the principal components of the reference K_{eq} tensor (see Section 2.2) out of the interval (0.5–2) are excluded from the study. This operation reduces the number of fields to 10,216.

Finally, the resolution of each image is augmented from 100×100 to 400×400 pixels in order to reduce the numerical error in the flow simulations that may be caused by connected components presenting a width inferior to 3 pixels.

The last test (Table 1, test 5) is done on 2196 3D binary fields of size $100 \times 100 \times 100$, obtained from an ensemble of micro-computerized-tomography (micro-CT) images of micro-metric sandstone, carbonate, synthetic silica and sand samples (Imperial College of London [30]). The aim of this test is not to give a credible $\mathbf{K}_{eq}$ estimation related to these materials, which should be done using pore-scale modeling techniques, but to have some initial