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Dynamic permeability of porous media by the lattice Boltzmann method

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ABSTRACT

The lattice Boltzmann method (LBM) is applied to calculate the dynamic permeability $K(\omega)$ of porous media; an oscillating macroscopic pressure gradient is imposed in order to generate oscillating flows. The LBM simulation yields the time dependent seepage velocity of amplitude A and phase shift B which are used to calculate $K(\omega)$. The procedure is validated for plane Poiseuille flows where excellent agreement with the analytical solution is obtained. The limitations of the method are discussed. When the ratio between the kinematic viscosity and the characteristic size of the pores is high, the corresponding Knudsen number Kn is high and the numerical values of $K(\omega)$ are incorrect with a positive imaginary part; it is only when Kn is small enough that correct values are obtained. The influence of the time discretization of the oscillating body force is studied; simulation results are influenced by an insufficient discretization, i.e., it is necessary to avoid using too high frequencies. The influence of absolute errors in the seepage velocity amplitude δA and the phase shift δB on $K(\omega)$ shows that for high ω even small errors in B can cause drastic errors in $Re[K(\omega)]$. The dynamic permeability of reconstructed and real (sandstone) porous media is calculated for a large range of frequencies and the universal scaling behavior is verified. Very good correspondences with the theoretical predictions are observed.

1. Introduction

Oscillating flows through porous media are of great interest in various areas of Physics and Engineering. Propagation of acoustic waves in porous media saturated by one or two immiscible fluids corresponds to very important applications such as seismic investigations in oil fields. Oscillating flows were considered by Biot [1,2] in order to study low- and high-frequency acoustic waves propagating through saturated porous media. Various types of waves propagating through the medium depending on the frequencies are predicted by this theory. Acoustic waves in statistically homogeneous saturated porous media can be also described by the homogenisation theory [3,4]; first, some local problems defined on the unit cell are solved; then, the corresponding solutions are averaged over the unit cell; finally, a generalised form of the Christoffel equation (see [5] for details) where these averages are present, is solved in order to obtain the acoustic velocities. To the best of our knowledge, these problems were not addressed by the method of spatial averaging devised by Whitaker [6].

In both cases, oscillatory flows in porous medium are characterized by the dynamic permeability $K(\omega)$ which is used to derive the acoustic properties of the medium; $K(\omega)$ is a complex number whose real and imaginary parts depend on the frequency ω. This concept of dynamic permeability is of interest mostly for liquids which have a significant inertia. In [7], the relationship between $K(\omega)$ and the geometry of the medium was studied for high and low frequencies and the length scale Λ characterising the dynamically connected pore size was introduced. The scaling behavior of $K(\omega)$ was studied in [8,9]; $K(\omega)$ can be considered as a scaling function with only two parameters; moreover, if the pore cross sectional area varies slowly, a universal behavior is observed which is independent of the porous medium. This scaling behavior was confirmed by numerical simulations [8] as well as experimental studies [9,10]. The influence of pore roughness on dynamic permeability was investigated in [11]. The symmetry of the dynamic viscous permeability tensor for spatially periodic structures was studied in [12].

Dynamic permeability was addressed for Non Newtonian fluids by Whitaker in a series of papers [13–15]. In addition, a stochastic theory was devised for poroelastic media by [16].

In order to determine $K(\omega)$ for Newtonian fluids, numerical calculations must be performed since analytical calculations are possible only for very simple geometries such as the Poiseuille flow [2]. The finite element method is used in [8] while the boundary element method is used in [17]. Cellular automata are applied to calculate $K(\omega)$ in [18]. Simulations of oscillating flows through samples of asphalt pavements and calculations of dynamic permeability by lattice Boltzmann method (LBM) can be found in [19,20].

In the present work, LBM is applied to simulate oscillating flows through spatially periodic porous media. This paper is organized as follows. In Section 2, the theoretical problem is described; a

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reconstruction procedure to generate spatially periodic porous media is recalled. In Section 3, the lattice Boltzmann model as well as the procedure to calculate the dynamic permeability are presented; the methodology is similar to the one used in [19,20]. The procedure is validated for Poiseuille flows. Then, the influence of the Knudsen number and of the oscillation frequency on the results is discussed; the role of the Knudsen number is carefully studied since no prior study of such influence could be found in the literature. In Section 4, the procedure is applied to calculate the dynamic permeability of reconstructed and real porous media. Some concluding remarks end this paper in Section 5.

2. General

2.1. Oscillating flows in porous media

In this section, the theoretical problem is described. The liquid is supposed to be Newtonian and obeys the incompressible Navier–Stokes equation

$$\rho \frac{D\mathbf{v}}{Dt} = \rho\mathbf{F} - \nabla p + \mu\nabla^2\mathbf{v}, \tag{1a}$$

$$\nabla \cdot \mathbf{v} = 0, \tag{1b}$$

where ρ is the fluid density, $\mathbf{v}$ the fluid velocity, $\mathbf{F}$ the body force, p the pressure, and μ the viscosity. The no slip boundary condition applies at the fluid–solid interface

$$\mathbf{v} = 0. \tag{2}$$

The force $\rho\mathbf{F}$ which is equivalent to a macroscopic pressure gradient $\overline{\nabla p}$, is assumed to be harmonic

$$\mathbf{F} = \hat{\mathbf{F}}e^{i\omega t}, \tag{3}$$

where ω is the wave frequency. As a direct consequence, the velocity and pressure fields are supposed to be of the form

$$\mathbf{v} = \hat{\mathbf{v}}(\mathbf{r})e^{i\omega t}, \quad p = \hat{p}(\mathbf{r})e^{i\omega t}, \tag{4}$$

where $\mathbf{r}$ is the position vector. Then, the Navier–Stokes equation can be rewritten in the linearized form

$$i\omega\rho\hat{\mathbf{v}} = \rho\hat{\mathbf{F}} - \nabla\hat{p} + \mu\nabla^2\hat{\mathbf{v}}, \tag{5a}$$

$$\nabla \cdot \hat{\mathbf{v}} = 0. \tag{5b}$$

When solved, this equation yields the local fluid velocity and the pressure. The macroscopic properties characterizing the oscillatory flow can be written as a dynamic Darcy law

$$\langle\hat{\mathbf{v}}\rangle = \frac{\mathbf{K}(\omega)}{\mu}\cdot\rho\hat{\mathbf{F}}, \tag{6}$$

where the brackets $\langle\rangle$ denote the volume average

$$\langle\circ\rangle = \frac{1}{\Omega}\int_\Omega \circ\, d\Omega, \tag{7}$$

where Ω is the volume of the domain. The dynamic permeability tensor $\mathbf{K}(\omega)$ is a complex valued tensor which depends on ω. For isotropic media, $\mathbf{K}(\omega)$ is spherical and equal to $K(\omega)\mathbf{I}$ where $\mathbf{I}$ is the unit tensor and $K(\omega)$ a complex number

$$K(\omega) = K_r(\omega) + iK_i(\omega) = |K(\omega)|e^{i\arctan(\frac{K_i}{K_r})} \tag{8}$$

when ω is zero, the imaginary part vanishes, and the usual permeability defined by the static Darcy law is obtained.

2.2. Porous media

2.2.1. Plane Poiseuille flow

The simplest configuration for the dynamic permeability calculation which can be considered as a primitive model of a porous medium is a two-dimensional plane channel. A fluid oscillates in the channel limited by the solid planes $y = 0$ and h because of an oscillating body force F_x along the x-axis. The dynamic permeability can be calculated analytically [2]. The velocity profile v_f along the y-axis is provided by the equation

$$\partial_t v_f = F_x + \nu\partial^2_{y^2} v_f, \tag{9}$$

which can be rewritten in the linearized form

$$i\omega\hat{v}_f = \hat{F}_x + \nu\partial^2_{y^2}\hat{v}_f, \tag{10a}$$

with the boundary conditions

$$\hat{v}_f(0) = 0, \quad \hat{v}_f(h) = 0. \tag{10b}$$

Then, the solution is given by

$$\hat{v}_f(y) = \frac{i\hat{F}_x}{\omega}\left[-1 + \frac{\cosh\left(\sqrt{\frac{i\omega}{\nu}}\frac{h-2y}{2}\right)}{\cosh\left(\sqrt{\frac{iw}{\nu}}\frac{h}{2}\right)}\right]. \tag{11}$$

The average velocity can be found by integration across the channel

$$\langle\hat{v}_f\rangle = \frac{1}{h}\int_0^h \hat{v}_f(y)dy = \frac{\hat{F}_x}{\omega^2}\left[-i\omega + \frac{2\sqrt{i\omega\nu}}{h}\tanh\left(\sqrt{\frac{iw}{\nu}}\frac{h}{2}\right)\right]. \tag{12}$$

The dynamic permeability is deduced from the Darcy law (6)

$$K = \frac{\nu}{\omega^2}\left[-i\omega + \frac{2\sqrt{i\omega\nu}}{h}\tanh\left(\sqrt{\frac{iw}{\nu}}\frac{h}{2}\right)\right] \tag{13}$$

It is easy to verify that the usual value of permeability is obtained when ω tends to zero

$$\lim_{\omega\to 0} K(\omega) = \frac{h^2}{12}. \tag{14}$$

2.2.2. Reconstructed porous media

More sophisticated porous media can be simulated. Spatially periodic reconstructed porous media can be generated by the reconstruction procedure described in [21]. The unit cell is composed of N_c elementary cubes of size a along each direction of space. The structure of the medium is described by a phase function $Z(\mathbf{x})$ which is equal to 1 if the point $\mathbf{x}$ belongs to the void, and 0 if the point belongs to the solid. The function is characterized by a porosity ε and a correlation function $R_Z(\mathbf{u})$

$$\varepsilon = \overline{Z(\mathbf{x})}, \quad R_z(\mathbf{u}) = \frac{\overline{[Z(\mathbf{x}) - \varepsilon][Z(\mathbf{x}+\mathbf{u}) - \varepsilon]}}{\varepsilon - \varepsilon^2}. \tag{15}$$

$R_z(\mathbf{u})$ only depends on the norm u of the translation vector $\mathbf{u}$ when the medium is isotropic. $Z(\mathbf{x})$ is obtained by thresholding standard Gaussian variables Y correlated by

$$R_Y(u) = e^{-\frac{u^2}{l_c^2}}, \tag{16}$$

where l_c is the correlation length. Therefore, the input parameters for the reconstruction algorithm are the unit cell size N_c, the porosity ε and the correlation length l_c.

3. Lattice Boltzmann method

The Navier–Stokes equation can be numerically solved by the lattice Boltzmann method (LBM) which is widely used to solve