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Shallow water SPH for flooding with dynamic particle coalescing and splitting

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ABSTRACT

In this paper an adaptive algorithm for Smoothed Particle Hydrodynamics (SPH) for the Shallow Water Equations (SWEs) is presented. The area of a particle is inversely proportional to depth giving poor resolution in small depths without particle refinement. This is a particular limitation for flooding problems of interest here. Higher resolution is created by splitting the particles, while particle coalescing (or merging) improves efficiency by reducing the number of the particles when acceptable. The new particle coalescing procedure merges two particles together if their area becomes less than a predefined threshold value. Both particle splitting and coalescing procedures conserve mass and momentum and the smoothing length of new particles is calculated by minimizing the density error of the SPH summation. The new dynamic particle refinement procedure is assessed by testing the numerical scheme against analytical, experimental and benchmark test cases. The analytical cases show that with particle splitting and coalescing typical convergence rates remain faster than linear. For the practical test case, in comparison to using particle splitting alone, the particle coalescing procedure leads to a significant reduction of computational time, by a factor of 15. This makes the computational time of the same order as mesh-based methods with the advantage of not having to specify a mesh over a flood domain of unknown extent a priori.

1. Introduction

Floods and tsunamis cause many casualties and huge economic losses, and numerical simulation is now an important part of flood risk analysis. To date, this kind of study has primarily been conducted using the shallow water approximation with classical Eulerian or grid-based numerical methods. To get reliable results in flood simulations, it is important to simulate both large and small-scale features accurately such as recirculation zones, shock waves, etc., while maintaining efficiency of the computation. In Eulerian models this is usually achieved using dynamically adaptive structured [1–3] or unstructured grids [4]. In the framework of Lagrangian models, the Smoothed Particle Hydrodynamics (SPH) technique offers an alternative approach and has recently been formulated for Shallow Water Equations (SWEs) with promising results: Ata and Soulamani [5] proposed a formulation to avoid the use of artificial viscosity; Rodriguez-Paz and Bonet [6] introduced an SPH-SWEs formulation derived from a variational approach; and de Leffe et al. [7] presented an SWEs scheme using the formulation proposed by Vila [8]. The aim of this work is to introduce the advantage of the adaptivity commonly adopted in Eulerian schemes for meshless SWEs Lagrangian models.

In the framework of SPH schemes, recent improvements have lead to the development of open boundary conditions [9], the modified virtual boundary method to impose closed boundary conditions on arbitrary geometries [10], and the discretization of the source terms [10,11] for irregular, discontinuous bathymetries. These improvements have enabled the SPH-SWEs schemes to simulate floods over initially dry, large domains [9]. In meshfree numerical schemes there have been some early attempts to introduce variable resolution for incompressible flow by either remeshing, particle splitting and particle insertion/removal techniques [12–18], recently Vacondio et al. [19] presented an adaptive SPH numerical scheme for Navier–Stokes Equations using particle splitting and coalescing. However, in the SWEs a time-varying iterative procedure is necessary to calculate the SPH smoothing length and density of each particle [20]. This makes the numerical scheme quite different from SPH solvers for the Navier–Stokes equations.

Particle splitting can provide increased resolution, but also leads to an ever increasing number of particles in the numerical domain. Without the ability to coarsen resolution and reduce the number of particles during a simulation, this is detrimental to the efficiency. In this work, we will present a 2-D particle

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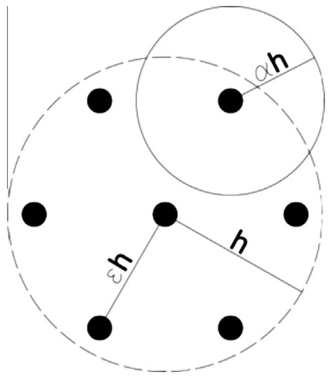

Fig. 1. Refinement patterns of one particle split into 7 daughter particles.

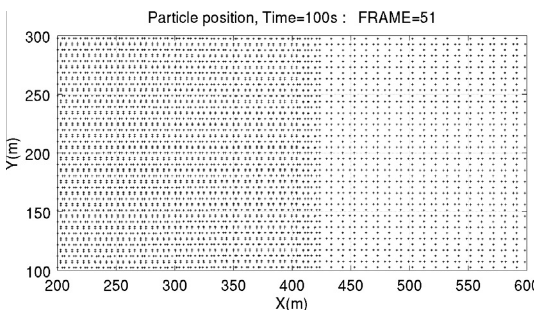

Fig. 2. Uniform flow in a sloping channel with friction, particle positions in an enlarged area of the coalescing region.

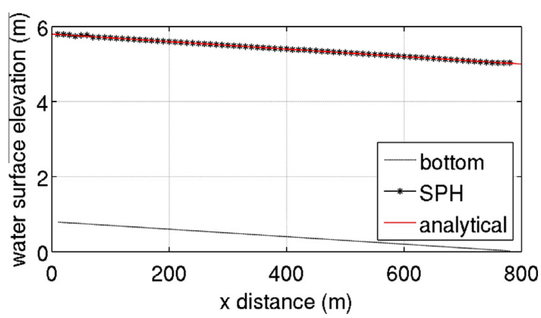

Fig. 3. Uniform flow in a sloping channel with friction, the water surface elevation at $y = 200$ m.

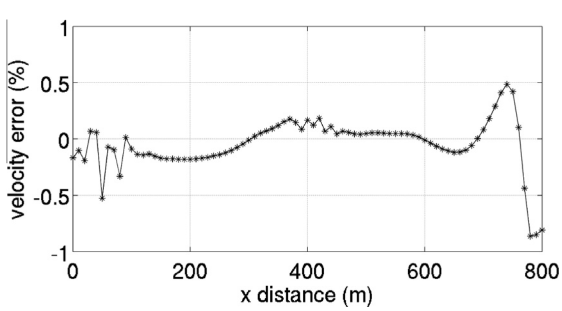

Fig. 4. Uniform flow in a sloping channel with friction, the velocity component v_x non-dimensional error at $y = 200$ m.

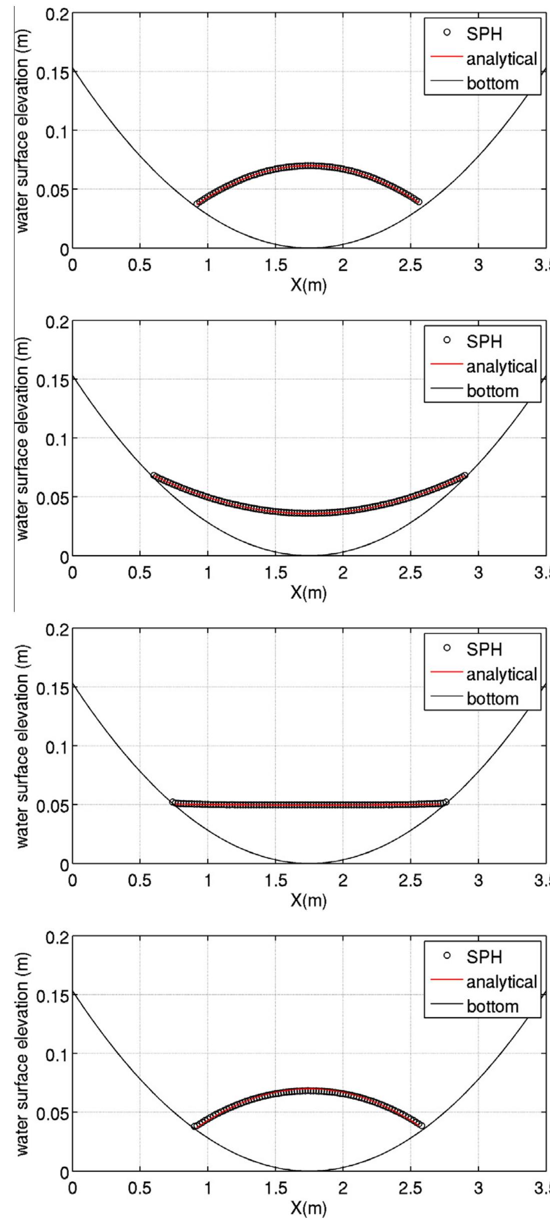

Fig. 5. Thacker's [26] test case, section at $y = 1.75$ m, water free surface elevation at times 0, 1.55, 2.44 and 3.15 s.

refinement scheme that includes particle splitting and coalescing (merging) giving an efficient SPH-SWEs numerical scheme. Both particle splitting and particle coalescing are derived to be consistent with the iterative procedure used to calculate the water depth.

This paper is organized as follows: in Section 2 the SPH-SWEs numerical method is presented together with the iterative procedure adopted to calculate the density and the smoothing length. In Section 3 the derivation of the particle splitting and coalescing procedures are presented. In Section 4 the numerical scheme is tested against analytical solutions for uniform flow in a sloping channel, an oscillating curved water surface in a parabolic basin and two dam-break experimental results. To assess the capability of the numerical scheme to simulate practical flood situations, the results of an hypothetical dam breach simulation over a real bathymetry have been compared with the results obtained by an Eulerian commercial code.

2. SPH-Shallow Water Equations with variable resolution

The Shallow Water Equations (SWEs) represent the depth-integrated equations of mass and momentum conservation and they can be written in Eulerian form as: