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Net beach change in the swash zone: A numerical investigation

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ABSTRACT

A range of bed-load sediment transport formulae are used to run fully coupled, morphodynamic simulations of one [1] swash cycle on an erodible plane beach. A system comprising shallow water equations and Exner equation is solved, in which sediment transport rate q is either dependent only on depth averaged velocity (u), or on u and water depth (h). The results are in agreement with equivalent uncoupled results [2] in that all sediment transport formulae considered applied to the event of [1] yield net erosion in the whole of the swash. Consistent with [3], however, full coupling yields significantly less erosion for all the $q = q(u)$ formulae compared to the equivalent uncoupled results. It is shown that differences between uncoupled and coupled approaches (for most formulae) accumulate over the course of the swash event. The main reason for the reduced net erosion is the smaller maximum inundation. It is also shown that including a dependence on h in the bed load sediment transport formula for fully coupled simulations can result in net deposition in the upper swash.

Bed shear stress described by a Chezy law is further included in fully coupled simulations to examine net beach change. Much reduced maximum inundation and net offshore sediment transport are predicted both for $q = q(u)$ and $q = q(h,u)$. It is shown that although the net sediment flux at the base of the swash under one [1] swash event is still offshore, deposition in the middle or upper swash may be predicted when bed shear stress is included, particularly when the drag coefficient in the backwash is reduced compared to that in the uprush, consistent with some in-situ measurements. The implication is that bed shear stress must be included not just to obtain correct quantitative beach change, but also to obtain correct qualitative beach behaviour.

1. Introduction

The swash zone is a very dynamic and complex regime of the nearshore, in which the beach face changes rapidly. On steeper beaches bore-driven swash is the dominant swash motion and significant progress in understanding bore behaviour has been made, e.g. [4–6]. These motions also have significant implications for planform evolution in the swash and its modelling [7,8].

Peregrine and Williams [1], henceforth PW01, generalized the shallow water solution in [6], for the shoreline motion and its near region, to the whole swash, leading to a dam-break problem on a sloping bed. Work by Guard and Baldock [9] has indicated that the PW01 solution is only a special case of the swash event in [6], as it neglects the momentum behind the bore. However, subsequent work by Pritchard [10] suggests that modifying the hydrodynamic boundary conditions does not make great qualitative difference to net sediment transport. Pritchard and Hogg [2] looked in detail at this event, in particular using depth-averaged velocity ($\hat{u}$) and flow depth ($\hat{h}$) from the PW01 analytical solution to provide predictions of instantaneous equilibrium (steady state) sediment transport and also bed-load transport over the swash event. Then, by integrating over the swash duration (at each location) they showed that a whole range of sediment transport formulae, when interpreted as instantaneous sediment fluxes that adjust immediately to change in flow, all yield qualitatively similar net offshore transport at every point in the swash. For net deposition to result in some regions of the swash (which it must sometimes if beaches are not continually to be eroded) it was necessary either to introduce a settling and entrainment lag, or to assume that sediment is pre-suspended in the surf zone.

Subsequently, Kelly and Dodd [3] considered a fully coupled model in which the flow equations and a bed evolution (Exner) equation with sediment transport formula $\hat{q} = A\hat{u}^3$ ($\hat{q}$ instantaneous sediment flux and A bed mobility parameter) are solved simultaneously, and the bed change thus allowed to feed back onto the swash hydrodynamics. Kelly and Dodd [3] showed that those same PW01 initial conditions on an initially plane but now erodible bed also lead to net erosion for the whole of the PW01 swash, but significantly less than that predicted by the uncoupled model [2]. In related work, qualitatively different bed profiles were obtained by Briganti et al. [11] for the equivalent coupled, (initially)

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flat mobile-bed dam-break problem for the $\hat{q} = A\hat{u}^3$ formula, and another equilibrium formula considered by Pritchard and Hogg [2], $\hat{q} = \overline{A}\hat{h}\hat{u}^3$ ($\overline{A}$ also bed mobility parameter). These results point to the possible importance of considering different sediment transport formulae when full morphodynamic coupling is implemented, to the extent that different equilibrium sediment transport and bed-load transport formulae may yield qualitatively different results from uncoupled predictions and from each other.

A further important physical mechanism affecting swash zone dynamics is bed shear stress. Its effect is particularly significant in the region of shallow water flow near the shoreline ([12,13]). To what extent the bed shear stress influences sediment transport in the swash, and whether a qualitatively different bed profile could be predicted when bed shear stress is included are of great interest and importance for the swash simulation.

Therefore in this work we examine a range of bed-load and equilibrium sediment transport formulae similar to those examined by Pritchard and Hogg [2] under the PW01 swash event in fully coupled simulations on a mobile, initially plane sloping bed. The sediment transport formulae examined are consistent with beaches where bed load transport is dominant, or on which suspended load transport occurs but with negligible scour and settling lag. The purpose is to examine the range of predictions for the different formulae, all of which are formulae in use in engineering models or extensions thereof, in fully coupled simulations, these being a true representation of morphodynamics on real beaches. Further, differences between fully coupled and uncoupled approaches are considered, as well as what this means for numerical modelling in the swash region (and also modelling of tsunami inundation). Importantly, a particular focus is whether the same event can yield net sediment movement of opposing signs depending on which sediment transport formula is used. Accordingly, we also examine the general dependence of net beach change in the swash via the sediment transport formula q, on powers of u and h. Lastly, bed shear stress, in the form of a Chezy drag law, is included in fully coupled simulations for the first time, to investigate the effects of bed friction on the beach face evolution, for different sediment transport formulae. The effect of varying drag coefficient between uprush and backwash is also examined.

In the next section we present the equations examined. We then state the sediment transport formulae to be examined in Section 3. In Section 4 the PW01 event simulations are presented. Finally, conclusions are arrived at.

2. Mathematical model

2.1. Governing equations

The system of equations governing 1D shallow water flow with a mobile bed is:

$$\hat{h}_{\hat{t}} + \hat{u}\hat{h}_{\hat{x}} + \hat{h}\hat{u}_{\hat{x}} = 0, \tag{1}$$

$$\hat{u}_{\hat{t}} + \hat{u}\hat{u}_{\hat{x}} + g\hat{h}_{\hat{x}} + g\widehat{B}_{\hat{x}} = 0, \tag{2}$$

$$\widehat{B}_{\hat{t}} + \xi\hat{q}_{\hat{x}} = 0, \tag{3}$$

where $\hat{h}$ represents water depth (m), $\hat{u}$ is a depth-averaged horizontal velocity (ms^{-1}), $\widehat{B}$ is the bed level (m), $\hat{q}$ is sediment flux (m^{2}s^{-1}), which is, in general, a function of $\hat{h}$ and $\hat{u}$, $\xi = \frac{1}{1-p}$ with p being bed porosity, and g is acceleration due to gravity (ms^{-2}). In Fig. 1 we illustrate the situation being considered.

2.2. Non-dimensionalization

To make the results more intercomparable, we non-dimensionalize all variables. Dimensionless variables are:

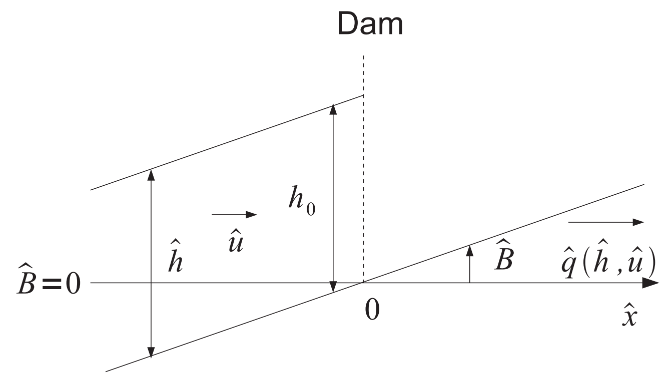

Fig. 1. Initial conditions for PW01 swash.

$$x = \frac{\hat{x}}{h_0}, \quad t = \frac{\hat{t}}{h_0^{1/2}g^{-1/2}}, \quad h = \frac{\hat{h}}{h_0}, \quad u = \frac{\hat{u}}{(gh_0)^{1/2}},$$
$$B = \frac{\widehat{B}}{h_0} \text{ and } q = \frac{\hat{q}}{q_0}, \tag{4}$$

where h_0 is a length scale, and q_0 represents a sediment flux scale.

Substituting (4) into the governing Eqs. (1) and (2) gives:

$$h_t + uh_x + hu_x = 0, \tag{5}$$

$$u_t + uu_x + h_x + B_x = 0. \tag{6}$$

Assuming $q = q(h, u)$ and substituting (4) into (3) gives:

$$B_t + \sigma q_h h_x + \sigma q_u u_x = 0, \tag{7}$$

where $\sigma = \frac{\xi q_0}{g^{1/2}h_0^{3/2}}$.

These equations can be written in vector form:

$$\vec{U}_t + \mathbf{A}(\vec{U})\vec{U}_x = 0 \tag{8}$$

with

$$\vec{U} = \begin{bmatrix} h \\ u \\ B \end{bmatrix}, \quad \mathbf{A}(\vec{U}) = \begin{bmatrix} u & h & 0 \\ 1 & u & 1 \\ \sigma q_h & \sigma q_u & 0 \end{bmatrix}.$$

The eigenvalues of $\mathbf{A}$ are the roots of the polynomial,

$$\lambda^3 - 2u\lambda^2 + (u^2 - \sigma q_u - h)\lambda + \sigma(uq_u - hq_h) = 0, \tag{9}$$

the roots of which may be denoted λ_1, λ_2 and λ_3, such that $\lambda_1 \leqslant \lambda_3 \leqslant \lambda_2$. When $\sigma \to 0$, one root tends to 0, and the other two tend to the corresponding hydrodynamic characteristic speeds.

2.3. Riemann equations and numerical solution

The specified time interval method of characteristics (STI MOC) which has been successfully used in [3,14] is employed. The resulting Riemann (characteristic) equations for a general formula $q(h, u)$ are:

$$R^{(k)} = \lambda_k \frac{du}{dt} + \frac{\lambda_k + \sigma q_h}{\lambda_k - u}\frac{dh}{dt} + \frac{dB}{dt} = 0, \quad k = 1, 2, 3, \tag{10}$$

which hold along the characteristics $\frac{dx}{dt} = \lambda_k$, with k defining the characteristic family. This method is used here to achieve very high accuracy in the vicinity of discontinuities (e.g. bores and hydraulic jumps), at which locations usual engineering codes lose some accuracy; see [15]. See [3] for further details of the numerical method.