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Stochastic joint inversion of hydrogeophysical data for salt tracer test monitoring and hydraulic conductivity imaging

A. Jardani [a,*], A. Revil [b,c], J.P. Dupont [a]

[a] Université de Rouen, M2C, UMR 6143, CNRS, Morphodynamique Continentale et Côtière, Bât. IRESE A, Mont Saint Aignan, France
[b] Colorado School of Mines, Dept. of Geophysics, Golden, CO, USA
[c] ISTerre, CNRS, UMR CNRS 5275, Université de Savoie, 73376 Cedex, Le Bourget du Lac, France

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ABSTRACT

The assessment of hydraulic conductivity of heterogeneous aquifers is a difficult task using traditional hydrogeological methods (e.g., steady state or transient pumping tests) due to their low spatial resolution. Geophysical measurements performed at the ground surface and in boreholes provide additional information for increasing the resolution and accuracy of the inverted hydraulic conductivity field. We used a stochastic joint inversion of Direct Current (DC) resistivity and self-potential (SP) data plus in situ measurement of the salinity in a downstream well during a synthetic salt tracer experiment to reconstruct the hydraulic conductivity field between two wells. The pilot point parameterization was used to avoid over-parameterization of the inverse problem. Bounds on the model parameters were used to promote a consistent Markov chain Monte Carlo sampling of the model parameters. To evaluate the effectiveness of the joint inversion process, we compared eight cases in which the geophysical data are coupled or not to the in situ sampling of the salinity to map the hydraulic conductivity. We first tested the effectiveness of the inversion of each type of data alone (concentration sampling, self-potential, and DC resistivity), and then we combined the data two by two. We finally combined all the data together to show the value of each type of geophysical data in the joint inversion process because of their different sensitivity map. We also investigated a case in which the data were contaminated with noise and the variogram unknown and inverted stochastically. The results of the inversion revealed that incorporating the self-potential data improves the estimate of hydraulic conductivity field especially when the self-potential data were combined to the salt concentration measurement in the second well or to the time-lapse cross-well electrical resistivity data. Various tests were also performed to quantify the uncertainty in the inverted hydraulic conductivity field.

1. Introduction

The steady-state ground water flow and solute transport are mainly controlled by the spatial distribution of the permeability and dispersivity of an aquifer. Permeability can vary over 12 orders of magnitudes and can be very heterogeneous at various scales, which in turn implies a complex pattern for groundwater flow and contaminant transports [1]. At the opposite, porosity and dispersivity does not exhibit such a broad range of variation. The hydraulic conductivity is most commonly estimated from invasive hydrogeological techniques, such as pumping tests. Despite recent advances in hydraulic tomography [2–4] the resolution of the inverted hydraulic conductivity depends strongly on the density of piezometers [5]. The limited number of available piezometers makes the reconstruction of the hydraulic conductivity field of heterogeneous aquifers a difficult problem in most practical cases [6]. The use of geophysical methods can provide additional complementary information as broadly acknowledged in the last decade [7,8].

Recently, geophysical tools have benefited from (i) the evolution of the efficiency of numerical methods (for instance the mixed finite element approach) for solving partial differential equations and parallel computing [9,10], (ii) the development of improved petrophysical models connecting the geophysical signature to the hydraulic properties [11–14], and (iii) significant improvements in the technology of various sensors and filtering techniques with the possibility to developed multitask sensors. These developments have therefore given birth to a new era of three-dimensional time lapse geophysical imaging for tracking the changes of variables of interest like the moisture content, the salinity, and the pore fluid pressure [7,9–12,15].

Along these lines, the electrical resistivity imaging (ERI) is sensitive to changes in pore water electrical conductivity and temperature and therefore it has been used to track the subsurface

* Corresponding author.
E-mail addresses: Abderrahim.jardani@univ-rouen.fr (A. Jardani), arevil@mines.edu (A. Revil), Jean-paul.Dupont@univ-rouen.fr (J.P. Dupont).

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migration of conductive tracers (saline or heat tracer tests) with the goal to image the hydraulic conductivity of heterogeneous aquifers [16]. Pollock and Cirpka [17] presented recently a new analysis of the ERI data to image the distribution of the hydraulic conductivity. Their work is based on the analysis of temporal moments of potential electrical disturbances recorded during saline tracer tests. Various geophysical methods with different sensitivity maps can be also used in concert. For instance, Direct Current (DC) resistivity data can be also jointly inverted with Ground Penetrating Radar (GPR) data during salt tracer tests in the vadose zone, to determine the hydraulic conductivity and petrophysical properties such as electrical formation factor, the water content, and the effective grain radius of the sediments [18,19].

In this paper, we are interested by looking at the value of adding self-potential (SP) measurements in the joint inversion of geophysical data and in situ salt tracer sampling. The goal is still the same: the inversion of the hydraulic conductivity of an heterogeneous aquifer between two wells. The self-potential signals are passively recorded electrical potential signals associated with the occurrence of natural (source) currents in the ground. In the case of a salt tracer test, the source current density is generated by two contributions (i) the gradient in the activity of the salt (diffusion current) and (ii) an electrokinetic coupling (streaming current) directly associated with the flow of the ground water. The occurrence of this second contribution has been recently used to non-intrusively assess ground water flow For instance, Jardani et al. [20] used SP data to reconstruct the hydraulic head variations associated with pumping tests conducted in an alluvial aquifer. Suski et al. [21] showed that SP signals can be used to monitor the variations of the piezometric levels of an unconfined aquifer associated with an infiltration experiment from a ditch. SP data have been combined with the hydraulic head variations recorded during pumping tests to estimate the transmissivity of an heterogeneous aquifer [22,23] and to image in 3D the hydraulic conductivity [24]. The SP responds associated with the diffusion source current is recognized as an efficient method to delineate contaminated areas [25] and salt tracer tests [26]. We feel that the time-lapse analysis of the self-potential signals associated with a salt tracer test could emerged as a powerful tool in determining quantitatively hydraulic properties.

As a side note, other geophysical methods have been conducted in order to evaluate the hydraulic conductivity of heterogeneous aquifers. For instance, Linde et al. [27] used cross-well ground penetrating radar (GPR) for salt tracer tests. Hyndman et al. [28] presented recently a relationship between the seismic slowness and hydraulic conductivity, which has been successfully used to predict the hydraulic conductivity of an alluvial aquifer from seismic and tracer test data. Hördt et al. [29] proposed the use of spectral induced polarization measurements to image the hydraulic conductivity of a sandy/gravel aquifer.

The big picture is that combining hydrogeophysical and hydrogeological information need to be somehow coupled to reduce the uncertainty associated with the estimation of the hydraulic conductivity. This implies to take into account the uncertainty associated with the in situ measurements as well as with the geophysical data. Two approaches can be used to combine hydrogeological and geophysical data. In the first approach, the hydrological information is used to weakly constrain the inversion of the geophysical measurements [27]. The second approach is to fully couple the inversion of the two types of data [7,17,30]. Hinnell et al. [31] devoted an approach using the coupling of hydrological and geophysical data to reconstruct the hydrological parameters using ERI for tracking the infiltration front in vadose zone in a synthetic case study.

In this paper, we are interested by the fully coupled joint inversion of geophysical and hydrogeological data to monitor a salt tracer test and to assess the hydraulic conductivity field. Using salt tracer tests with ERI is a problem that has been received a lot attention recently [32–37]. The non-uniqueness of the ERI inverse problem (and its sensitivity map) makes this method insufficient by itself. In other words, ERI needs to be combined with other sources of information like in situ measurements of the salt concentration in wells [33,38,39]. Irving and Singha [38] introduced a stochastic joint inversion approach of time-lapse cross-well ERI and salt tracer concentration data. Our work is following this idea but adding an additional method, the self-potential method, to the inverse problem. We use the pilot points approach for the joint inversion of ERI, SP, and salt tracer data because this approach reduces the over parametrization of the problem. While previous authors used deterministic methods to choose the values of the pilot points, we place the pilot points on a regular grid and we use a stochastic method based on the Markov chain Monte Carlo, McMC, sampling approach to estimate the values of the model parameters at the pilot points.

2. Theory

We present in this section the equations governing the physical processes of groundwater flow and saline tracer transport in a heterogeneous unconfined aquifer. We also introduce the semi-coupled equations connecting the salt concentration to the electrical resistivity (to interpret ERI) and to the source current density used to interpret SP data.

2.1. 2D flow and transport equations

In steady-state conditions, the governing groundwater flow equation in a saturated and heterogeneous porous material is given by the following elliptic partial differential equation:

$$\nabla \cdot (K \nabla h) = 0, \tag{1}$$

subject to the boundary conditions,

$$h = h_0 \quad \text{at} \quad \Gamma_D \tag{2}$$

$$-\hat{\boldsymbol{n}} \cdot K \nabla h = q_0 \text{ at } \Gamma_N, \tag{3}$$

where h denotes the hydraulic head (in m), K is the hydraulic conductivity (m s^{-1}, we assume that the aquifer is isotropic). Eqs. (2) and (3) correspond to Dirichlet and Neumann boundary conditions, respectively. The hydraulic head h_0 denotes the head fixed at the boundary Γ_D, q_0 is the hydraulic flux (m^2 s^{-1}) assumed to be known at the Neumann boundary Γ_N, and $\hat{\boldsymbol{n}}$ is the unit vector normal to the boundary Γ_N.

The constitution of the transport equation of a salt tracer consists in the coupling of the Darcy's law for the Darcy velocity $\mathbf{u}$ (in m s^{-1}), and Fick's law for the flux of the salt $\mathbf{j}_d$ (in kg m^{-2} s^{-1}) [40]:

$$\mathbf{u} = -K \nabla h, \tag{4}$$

$$\mathbf{j}_d = -\rho_f \phi \mathbf{D} \cdot \nabla c + \rho_f c \mathbf{u}, \tag{5}$$

where $\mathbf{D}$ (in m^2 s^{-1}) denotes the hydrodynamic dispersion tensor, ϕ (unitless) denotes the connected porosity, c (unitless) denotes the solute mass fraction, and ρ_f (in kg m^{-3}) represents the solute bulk density.

The transport of the salt due to the injection of a salt tracer in the aquifer follows the advection–dispersion equation derived from a combination of the continuity equation (mass conservation equation for the salt) and the generalized Fick's law given by Eq. (5):

$$\frac{\partial(\rho_f \phi c)}{\partial t} + \nabla \cdot \mathbf{j}_d = 0. \tag{6}$$