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Application of a simple power law for transport ratio with bimodal distributions of spherical grains under oscillatory forcing

Kevin Holway [a,1], Christopher S. Thaxton [a,*], Joseph Calantoni [b,2]

[a] Department of Physics and Astronomy, Appalachian State University, 525 Rivers Street, Boone, NC 28608, USA
[b] Marine Geosciences Division, Naval Research Laboratory, Stennis Space Center, MS, USA

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ABSTRACT

Morphodynamic models of coastal evolution require relatively simple parameterizations of sediment transport for application over larger scales. Calantoni and Thaxton (2008) [6] presented a transport parameterization for bimodal distributions of coarse quartz grains derived from detailed boundary layer simulations for sheet flow and near sheet flow conditions. The simulation results, valid over a range of wave forcing conditions and large- to small-grain diameter ratios, were successfully parameterized with a simple power law that allows for the prediction of the transport rates of each size fraction. Here, we have applied the simple power law to a two-dimensional cellular automaton to simulate sheet flow transport. Model results are validated with experiments performed in the small oscillating flow tunnel (S-OFT) at the Naval Research Laboratory at Stennis Space Center, MS, in which sheet flow transport was generated with a bed composed of a bimodal distribution of non-cohesive grains. The work presented suggests that, under the conditions specified, algorithms that incorporate the power law may correctly reproduce laboratory bed surface measurements of bimodal sheet flow transport while inherently incorporating vertical mixing by size.

1. Introduction

The modeling of morphodynamic changes in the nearshore has historically focused on predicting changes in bed elevation driven by local spatial gradients in sediment transport [1,2]. However, even in the absence of bulk transport gradients, sediment grain sorting by size is ubiquitous. Grain size-dependent transport and the surficial distribution of sediments by size may play a critical role in determining bed friction for circulation models, in addition to influencing bedform and channel development, evolution, and migration. Recent attempts to incorporate fractional transport schemes into coupled wave-current-sediment models for the bottom boundary layer divide the bed into a user-defined number of size classes where conventional bedload transport formulae are computed using a median grain size (D_{50}) for each class [3]. Others employ stochastic models for rivers with dunes [4,5]. Although such attempts have demonstrated skill, they do not incorporate the effects of dynamic granular sorting during bedload transport resulting in vertical gradation of sediment by size within in the bed.

A simple, predictive power-law,

$$\frac{q_L}{q_S} = K\left(\frac{M_L}{M_S}\right)^{\alpha}, \tag{1}$$

was developed [6] using results obtained from a discrete particle model (DPM) [7] for the bed-parallel transport rates of grains by size in a bimodal distribution of coarse sediments under sheet flow (or near sheet flow) conditions given forcing from idealized monochromatic waves characteristic of the surf zone. Here, M_L and M_S are the local mass abundances (kg) of large and small grains and q_L and q_S are the horizontal transport rates (kg m^{-1} s^{-1}) of the large and small grains, respectively. The suggested values from simulations for the coefficients are $K = 5.34$ and $\alpha = 1.47$ [4]. The power law (1) results from the fact that, through direct simulation of the granular dynamics, the DPM reproduces, at high-resolution, rapid vertical kinematic sorting of grains by size that has been observed in the laboratory [8–10]. The power-law was used to accurately hindcast the relative transport rates from previous laboratory studies [9,10] with bimodal size ratios as high as 4.6:1 and having fluid forcing conditions similar to but not identical to the simulations.

Here, we have incorporated the power law (1) into a two-dimensional cellular automaton, BEDCELL, to simulate sheet flow transport. Unlike traditional solvers of advection-diffusion partial differential equations that transport residual mass based on differential relationships, or fractional transport approaches that do the

* Corresponding author. Tel.: +1 (0) 828 262 6836; fax: +1 828 262 2049.
E-mail addresses: kh73432@appstate.edu (K. Holway), thaxtoncs@appstate.edu (C.S. Thaxton), joe.calantoni@nrlssc.navy.mil (J. Calantoni).
[1] Tel.: +1 (0) 828 262 3090; fax: +1 828 262 2049.
[2] Tel.: +1 (0) 228 688 4435; fax: +1 228 688 4853.

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same for each fraction independently, BEDCELL transports all mass dictated by the net and fractional transport rates within the active layer while incorporating the dynamic vertical sorting implicit in (1). In addition, BEDCELL tracks vertical mass distributions per size fraction, producing sedimentation layers and/or eroding existing layers depending on the temporal characteristics of the fluid forcing. BEDCELL is designed to function for small temporal and spatial scales; however, it is numerically efficient and may readily be adapted to support larger-scale modeling than is presented here.

Herein, we validate BEDCELL for sheet flow conditions in which a bed comprised of a bimodal size distribution of noncohesive grains is driven by a monochromatic oscillatory forcing producing a uniform net transport rate. Initial and boundary conditions, as well as model control parameters, are set to match a set of experiments performed in the small oscillatory flow tunnel (S-OFT) at the Naval Research Laboratory at the Stennis Space Center, MS. The time-dependent surface sediment distribution measured in the S-OFT experiments compared well with that computed by BEDCELL. Our results suggest that, under the conditions specified, algorithms that incorporate the power law may correctly reproduce laboratory bed surface measurements of bimodal sheet flow transport while inherently incorporating vertical mixing by size.

2. Methods

2.1. Experimental facility

Experiments were performed in the small oscillatory flow tunnel (S-OFT) at the Naval Research Laboratory at the Stennis Space Center, MS, in which sheet flow transport was generated with a bed composed of a bimodal size distribution of non-cohesive grains. The sediment grains were nylon (material density 1170 kg m^{-3}) cylinders having a height equal to their diameter, D = 1.0 mm (white) and D = 0.5 mm (red). Sediments will be referred to as large (1.0 mm) and small (0.5 mm) for the remainder of the manuscript. The S-OFT was filled with fresh water and the temperature was nominally 25.6 °C during the experiments.

The S-OFT has a 200 cm-long test section with a 25 cm-wide channel. The upper portion of the sediment well was filled to a depth of 10 cm with the nylon grains. The entire bed was filled with small grains except for a 25 cm-wide rectangular "plug" of large grains located 100 cm from the "downstream" end of the S-OFT and 75 cm from the "upstream" end of the S-OFT (Fig. 1). The bed was smoothed to achieve a flat surface prior to the experiments.

Flow in the S-OFT is driven by a piston connected to a flywheel with a slider and connecting rod. The stroke length of the piston was chosen to produce an excursion amplitude in the test section of 8.74 cm. The variable frequency inverter was set to produce a period of 2.4 s. Under these forcing conditions, the maximum free stream velocity u_0 reached 23.2 cm/s, and the inherent asymmetry in acceleration produced a net sediment transport toward the piston in the "downstream" direction. The piston was turned on and run for about 25 periods (60 s) before being shutdown. The piston reaches the set frequency within two complete revolutions after start up.

A Canon™ EOS 7D digital SLR camera outfitted with a fisheye lens was mounted 100 cm directly above the center point of the bed surface, with the tangential plane of the lens parallel to the bed surface. High definition video of the sediment bed was acquired in 1080 p (1920 × 1080 pixels) at 30 fps. The camera captured 72 frames/period for a total of 1800 frames over 25 periods (60 s). The camera field of view at the bed was roughly 120 cm centered along the 200 cm test section (16 pixels/cm). The camera was calibrated with a standard checkerboard calibration sheet laid atop

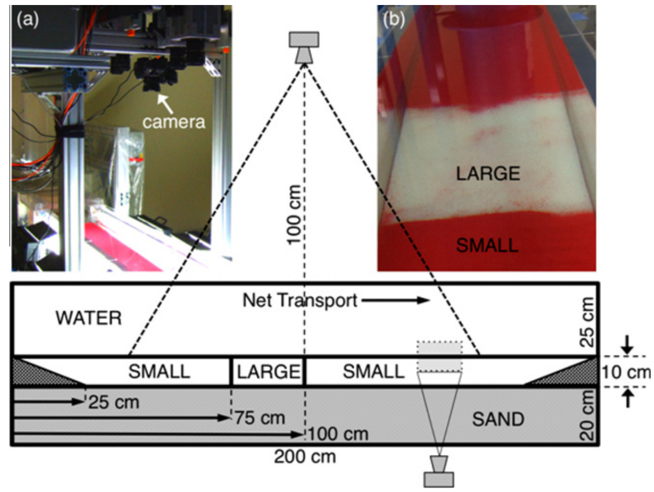

Fig. 1. Shown is a side view, schematic of the test section in the S-OFT drawn to scale. The total length of the test section is 200 cm. The sediment well is 30 cm deep. The lower 20 cm of the sediment well was filled with sand. The upper 10 cm was filled with the nylon material where the initial locations of the SMALL (red) and LARGE (white) grains are indicated. The ramps denoted at the ends of the test section were constructed of cobble filled with sand to minimize fluid turbulence. The dashed lines from the camera location above the test section denote the extent of the field of view on the bed. The light gray window in the right half of the test section denotes the field of view of the camera when looking through the sidewall. Shown in the inset (a) is a photo of the camera array mounted above the S-OFT test section with the camera depicted in the schematic labeled. Shown in inset (b) is a photo of the initial bed surface taken through the acrylic lid of the S-OFT.

the bed with the S-OFT filled and sealed. Images from the fisheye lens were remapped to a rectilinear projection prior to processing.

In a second experiment, the same Canon™ EOS 7D digital SLR camera with a standard zoom lens was used to image through the acrylic sidewall of the S-OFT such that the tangential plane of the camera lens was parallel to the sidewall. An identical sediment bed was constructed and identical piston forcing was used while high-definition video was acquired in 720p at 60 fps. The field of view (indicated in Fig. 1) had spatial dimensions of 15.4 cm wide by 10.1 cm tall (83 pixels/cm).

2.2. Numerical methods

BEDCELL is a 2-D automaton that resolves longitudinal and vertical sorting of grains by size during bedload transport. Input to BEDCELL includes forcing and bed information. Forcing input can currently take two forms: (1) a time series of net transport rates (kg m^{-1} s^{-1}) along the longitudinal domain and (2) a time series of wave statistics from which net transport rates along the longitudinal domain can be derived from commonly-used bedload transport formulae (e.g., Meyer-Peter, Bagnold, van Rijn, Yalin, Madsen and others).

The user initializes the bed by defining an overall bed mass, longitudinal length, and depth. The bed is then segmented it into any number of columns of fixed longitudinal length Δx. All columns can be segmented into multiple vertical layers of fixed or varying depth, resulting in a 2-D matrix of cells. Each layer within individual columns can be initialized to any grain size distribution quantified through a mean concentration and concentration profile as described below.

Instead of solving advection-diffusion partial differential equations (PDEs), BEDCELL models transport through advection and diffusion cellular algorithms that require less computational resources than full PDE solvers for very large spatial and temporal scales. As such, BEDCELL propagates mass with a computational velocity of one longitudinal column per iteration – the net