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Lagrangian multiphase modeling of sand discharge into still water

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ABSTRACT

Discharge of sand into water can be modeled as the multiphase flow of a non-Newtonian and a Newtonian fluid by treating the granular material as a continuum. The numerical modeling of this article is on the basis of the latest generation of computational methods, the mesh-free Lagrangian (particle) methods. In these methods, the solution domain is discretized by a set of nodes or particles, possessing the field variables and moving in a Lagrangian coordinates. This makes these methods the powerful tool for handling any deformation or fragmentation in interfaces, which is a usual problem in multiphase granular flow (e.g., sand–water systems). The Moving Particle Semi-implicit method (MPS) is used in this study. A multiphase non-Newtonian MPS approach is developed and applied to the case of sand discharge into still water. The results are validated using experimental measurements and analytical solutions to evaluate the accuracy of the model. The effects of flow conditions and rheological properties on the behaviour of sand discharge into water are also investigated.

1. Introduction

Understanding the characteristics of the two-phase system of sand discharge into water is important for activities such as pumping of tailing into settling tanks, dredging and island building operations, and discharge of industrial and urban wastes. A number of numerical and experimental studies have been conducted on this subject. Jiang et al. [1] conducted experiments and numerical simulations for predicting the settling dispersion process of fine sediments discharged into water. Jiang et al. [2], Mazurek et al. [3], and Hall et al. [4] conducted measurements of the properties (e.g., concentration and velocity profiles) of a jet of sand and slurries discharged into still water.

For decades, the numerical simulation of multiphase flows (e.g., multiphase sand–water systems) has been on the basis of the conventional mesh-based methods (either Eulerian or Lagrangian) such as finite volume methods (FVM) and finite element (FEM) (e.g., [5,1]). Mesh-based numerical approaches can encounter some major issues, handling complicated phenomena with the large interfacial fragmentation or deformation, which is usual in the multiphase sand–water systems. Eulerian mesh-based methods are not inherently able to capture the interfaces and Lagrangian mesh-based methods have the issue of the adaptability connectivity. Additional interface tracking and positioning schemes, such as the marker-and-cell (MAC) (e.g., [5]), made the Eulerian mesh-based method able to handle the interfacial deformations. However, even with use of such schemes, the methods still have issues in the maintenance of very sharp or fragmented interfaces [6] and also require some complicated treatments for the cells containing the interface.

A recent strong interest in the area of numerical modeling focuses on a new group of numerical methods, the mesh-free Lagrangian (particle) methods, which are expected to produce better results than the conventional mesh-based methods in many applications [7]. Mesh-free Lagrangian methods deploy a set of discrete elements, called particles (with no connectivity), to represent the continuum and record the state and motion of the flow system in a Lagrangian coordinates [7]. Being free from the issues related to mesh systems (e.g., mesh generation, treatment and adapting), these approaches are capable to handle a broad verity boundary and interface deformations and fragmentations. The beginning of the mesh-free particle methods dates back to 1977, with Lucy [8] and Gingold and Monaghan [9] who developed the Smoothed Particle Hydrodynamics (SPH) method. SPH is based on the idea of replacing the fluid with a set of moving particles and transforming the governing PDEs into the kernel estimate integrals [7]. This method was first developed to solve astrophysics problems and was later extended to problems of continuum mechanics. Molecular dynamics [10,11] is another particle method developed for micro-scale applied mechanics problems. Some other popular particle methods are the Dissipative Particle Dynamics (DPD) [12], the Moving Particle Semi-implicit method (MPS) [13] and the Lattice-Boltzmann Method (LBM) [14].

Present study is on the basis of the MPS method. MPS is a deterministic mesh-free Lagrangian method originally developed for simulations of incompressible free-surface viscous flows. The method

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uses the weighted averaging process to approximate derivatives and physical properties. Over the past few years, the MPS method has been applied to a verity of flow problems such as breaking wave [15,16], dam break [17–20], flow over spillways [21], and flow over sills and trenches [20,22]. Shakibaeinia and Jin [19] proposed the weakly-compressible MPS method (WC-MPS) for simulation of incompressible fluids. They extended their model to multiphase flow and applied it to two-phase Poiseuille flow, Rayleigh–Taylor instability and Kelvin–Helmholtz instability [23] and multiphase dam-break [18]. Gotoh and Fredsøe [24] proposed a Newtonian multiphase MPS model for a solid–liquid two-phase system. Considering rheological properties, Shakibaeinia and Jin [18] proposed a non-Newtonian multiphase WC-MPS model for simulation of granular flow and then applied it to the a dam break problem with movable bed material.

This study aims to apply and evaluate the newly-developed non-Newtonian multiphase WC-MPS method for studying the sand discharge into still water, which is a system of multiphase granular flow in the grain inertia regime of granular flow. Unlike the Gotoh and Fredsøe [24] model, here, the rheological behavior of the granular flow is considered by treating the sand phase as a non-Newtonian fluid. The results are validated using analytical and experimental data. The model in this study is expected to reproduce the complicated behavior of a turbulent two-phase sand–water system. Effects of the flow conditions and rheological properties on the flow features of sand discharge into water are also addressed.

2. Numerical model

2.1. Governing equations

Considering the continuum description of the granular material, the conservation of mass and momentum can be used to describe the multiphase system of sand and water flow. The governing equations in the Lagrangian coordinates can then be given by

$$\frac{D\rho}{Dt} + \rho(\nabla \cdot \mathbf{u}) = 0, \tag{1}$$

$$\rho\frac{D\mathbf{u}}{Dt} = -\nabla p + \nabla(\mu\nabla \cdot \mathbf{u}) + \mathbf{F}, \tag{2}$$

where ρ is the fluid density, $\mathbf{u}$ is the flow velocity vector, p is the pressure value, μ is the dynamic viscosity, and $\mathbf{F}$ represents the body forces (e.g., gravity). In the Lagrangian coordinates, there is no special acceleration term in the mass and momentum conservations, and the motion of particles is directly given by $D\mathbf{r}/Dt = \mathbf{u}$, ($\mathbf{r}$ is the position vector). Unlike the regular multiphase models, here a single set of flow equations is employed to all of the phases and multiphase behavior is automatically reproduced by considering the system as single-phase, multi-density, multi-viscosity flow.

Here, the weakly-compressible MPS, WC-MPS method [19], is applied to calculate the pressure. The fluids are considered as nearly incompressible; therefore, an equation of state (EOS) can be employed to explicitly determine the pressure value for each of the particles. The Tait's EOS applied by Shakibaeinia and Jin [19] for the MPS method is given by

$$p_i^{n+1} = \frac{\rho_i c_0^2}{\gamma}\left(\left(\langle n^*\rangle_i/n^0\right)^{\gamma} - 1\right), \tag{3}$$

where c_0 is a numerical sound speed (an artificial value). A common value of $\gamma = 7$ is used here. Note that, c_0 usually has a value much smaller than the real sound speed of the fluid and is calculated based on the allowable density variation. For instance, one can use a c_0 around ten times of flow bulk velocity to maintain the density changes at less than one percent. c_0 is kept constant in the whole simulation.

2.2. MPS approximations

MPS formulation is based on the local weighted averaging of quantities and derivatives. Similar to other particle methods, in MPS method the continuum (i.e. the fluids) is represented by a particle system, where, each particle possesses a set of physical quantities (e.g., pressure and velocity) and moves according to the material velocity. A particle i with position vector $\mathbf{r}_i$, interacts with a particle j in using a weight (kernel) function, $W(r_{ij}, r_e)$, in which, $r_{ij} = |\mathbf{r}_j - \mathbf{r}_i|$ is the distance of particle j from particle i, and r_e is the radius of interaction (support) area around each particle, within which that particles affected by other particles. Here, the 3rd order polynomial spiky weight function [19] is applied. Density of particles around a specific particle is represented by a dimensionless parameter, referred as the particle number density, n, given by

$$\langle n\rangle_i = \sum_{j\neq i} W(r_{ij}, r_e), \tag{4}$$

with $\langle\rangle$ being the weight averaging operator. The pressure gradient term, ∇p, between the target particles i and its neighboring particles j is approximated by

$$\langle\nabla p\rangle_i = \frac{d}{n^0}\sum_{j\neq i}\left(\frac{p_j - \bar{p}_i}{r_{ij}}\mathbf{e}_{ij}W(r_{ij}, r_e)\right); \quad \mathbf{e}_{ij} = \frac{\mathbf{r}_{ij}}{r_{ij}}, \tag{5}$$

where n^0 is the initial particle number density and d is the number of space dimensions. $\underset{i}{\widehat{p}}$ is defined by

$$\underset{i}{\widehat{p}} = \min_{j\in J}(p_i, p_j); \quad J = \{j : W(r_{ij}, r_e) \neq 0\}, \tag{6}$$

The Laplacian of the velocity vector $\mathbf{u}$ on each particle is expressed by

$$\langle\nabla^2\mathbf{u}\rangle_i = \frac{2d}{\lambda n^0}\sum_{j\neq i}((\mathbf{u}_j - \mathbf{u}_i)W(r_{ij}, r_e));$$
$$\lambda = \int_V W(r, r_e)r^2 dv \Big/ \int_V W(r, r_e)dv, \tag{7}$$

Considering the multiphase force due to the discontinuity in the viscosity field, the MPS formula of the divergence-free viscous term suggested by Shakibaeinia and Jin [18] is given by

$$\langle\nabla(\mu\nabla \cdot \mathbf{u})\rangle_i = \frac{4d}{\lambda n^0}\sum_{j\neq i}\left(\frac{\mu_i\mu_j}{\mu_i + \mu_j}(\mathbf{u}_j - \mathbf{u}_i)W(r_{ij}, r_e)\right), \tag{8}$$

2.3. Granular rheology

Sand–water mixture is a solid–liquid two-phase (including water phase and sand phase) system. Similar to water phase, the sand phase considered as a continuum. Calculating the viscous terms, the dynamic viscosity of each of the particles needs to be determined. Determining the viscosity of the sand phase is not as straightforward as the water phase with a constant viscosity. One can calculate the viscosity of the sand phase simply by treating it as a Newtonian fluid (e.g., the method used by Gotoh and Fredsøe [24]). However, the complicated behavior of the granular flow cannot correctly be predicted using a simple Newtonian model; therefore, a rheological model is required. The rheological model used here is the Bingham plastic model, which has widely been employed for granular flows [25]. In the Bingham plastic model, the material starts to flow as a viscous fluid only if stresses exceed a yield stress, τ_y, otherwise it behaves like a rigid body. The shear stress in the Bingham plastic model can be calculated by