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De Saint-Venant equations-based model assessment in model predictive control of open channel flow

M. Xu [a,*], R.R. Negenborn [b], P.J. van Overloop [a], N.C. van de Giesen [a]

[a] Section of Water Resources Management, Faculty of Civil Engineering and Geosciences, Delft University of Technology, Stevinweg 1, 2628CN, Delft, The Netherlands
[b] Section of Marine and Transport Technology, Faculty of Mechanical, Maritime and Materials Engineering, Delft University of Technology, Mekelweg 2, 2628CD, Delft, The Netherlands

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ABSTRACT

Model predictive control (MPC) is a model-based control technique that uses an optimization algorithm to generate optimal control actions. Based on the model used in optimization, MPC approaches can be categorized as linear or nonlinear. Both classes have advantages and disadvantages in terms of control accuracy and computational time. A typical linear model in open channel water management is the Integrator Delay (ID) model, while a nonlinear model usually refers to the Saint-Venant equations. In earlier work, we proposed the use of linearized Saint-Venant equations for MPC, where the model is formulated in a linear time-varying format and time-varying parameters are estimated outside of the optimization. Quadratic Programming (QP) is used to solve the optimization problem. However, the control accuracy of such an MPC scheme is not clear. In this paper, we compare this approach with an MPC scheme that uses Sequential Quadratic Programming (SQP) to solve the optimization problem. Because the estimation of the time-varying parameters is integrated in the optimization in SQP, the solutions from SQP-based MPC are expected to be superior to the solutions of QP-based approach. However, SQP can be computationally expensive. A simulation experiment illustrates that the QP-based MPC approach using a linearized Saint-Venant model has an accurate approximation of the control performance of SQP.

1. Introduction

Over the last decade, model predictive control (MPC) of open channel flow has been a subject of extensive study [1–6]. MPC is a model-based control technique that uses an optimization algorithm to generate optimal control actions. Advantages of MPC are that it predicts the future system dynamics, therefore being able to take into account future known disturbances. It can also deal with constraints within the optimization. Based on the type of model used in the optimization, MPC approaches can be categorized as linear or nonlinear. The focus of MPC in open channel water management is mainly on efficient water delivery in irrigation systems, and river operations for flood or drought prevention. A common feature of the existing research is that, typically, linear models are used for predicting the system dynamics, such as the reservoir model and the classical Integrator Delay (ID) model in [1–5]. Under certain assumptions, these linear models can approximate the nonlinear system dynamics well. The MPC optimization problems when using such linear models are easy and fast to solve. Moreover, guaranteed global optimal solutions can be found.

A nonlinear model can normally include more system dynamics than a linear one. This extra information in the nonlinear model may increase the control accuracy in MPC. However, due to the use of such a nonlinear model, the optimization problem can become non-convex and hard to solve. Indeed, this is the case when using the Saint-Venant equations. Theoretically, a guarantee for finding the global optimum for nonlinear optimization cannot often be given [7]. Since the optimal action needs to be taken within a prescribed time period in real-time control, computational time is important in achieving the optimum. Unfortunately, such a nonlinear MPC scheme can be very time consuming, e.g., due to the CPU-intensive model executions for the numerical calculation of gradients of a Lagrangian function with respect to the control variables, especially in the areas where these gradients are flat. This computational complexity in MPC using such a nonlinear model was also stated by Barjas Blanco [6]. Therefore, they used a series of reservoir models instead.

Some researchers use adjoint sensitivity analysis to speed up the nonlinear optimization by analytically calculating the gradients of the Lagrangian function with respect to the control variables [8,9]. This is attractive for making such an MPC implementation feasible in real-time control, but it needs extensive analytical analysis of the nonlinear model and its derivatives beforehand. Moreover, any change to the control problem requires a new analytical derivation. For these reasons, the

* Corresponding author. Tel.: +31 0 152782345; fax: +31 0 152785559.
E-mail address: min.xu@tudelft.nl (M. Xu).

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adjoint sensitivity analysis is not conducted in this paper. Instead, in [10], we proposed an MPC scheme using linearized Saint-Venant equations in a time-varying format to approximate the nonlinear dynamics. The scheme requires a much more complex discretization and mathematical formulation of the equations than the reservoir model in [6]. This MPC scheme solves the optimization problem with a standard Quadratic Programming (QP) solver, which considers the model constraints as linear.

The MPC approach in [10] is found to be the most accurate for comparison with MPC using an Integrator Delay model and a Reduced Saint-Venant model. The proposed method formulates the Saint-Venant equations as a linear time-varying state-space model. It uses a 'Forward Estimation' to estimate the time-varying parameters outside of the optimization, based on the optimal solutions over a prediction horizon from the previous control step. However, due to the lack of information at the last prediction step, the optimal solutions in the previous control step are not optimal anymore in the present step. Therefore, it is unclear what the performance of this QP-based MPC controller is. The purpose of this work is to explore the accuracy of the control procedure of [10] by comparing the results with an MPC controller that formulates Sequential Quadratic Programming (SQP) problems and solves the entire time-varying Saint-Venant equations within the optimization. According to Schittkowski [11], SQP is a state-of-the-art method for solving nonlinear programming problems. Here the MPC scheme using this method is called SQP-based MPC.

In this paper, we focus on the performance assessment of the two MPC schemes in terms of water level deviations from the target and the control actions. Because of the integrated calculation of the time-varying parameters within the optimization, the solutions from SQP-based MPC are expected to be superior to the solutions of QP-based approach, given sufficient computation time. It is the question how the two methods compare in terms of computational time and control accuracy. Regarding the control accuracy, the SQP-based MPC can be used as a benchmark. In addition, for the QP-based MPC, iterations are added between the 'Forward Estimation' and the 'Quadratic Programming' blocks, in order to compensate the influence of the lack of information at the last prediction step. Therefore, another goal of this work is to investigate the significance of this influence.

This paper is organized as follows. Section 2 describes the main components of MPC, including the open channel flow modeling and the optimization problem formulation. It summarizes the QP-based MPC scheme using the linearized Saint-Venant model and introduces the SQP-based MPC scheme. Section 3 introduces the test case used to compare the control performance between the two MPC schemes. A detailed demonstration of the results is given in Section 4 and conclusions and future research are given in Section 5.

2. Model predictive control of open channel flow

Model predictive control has a general structure which uses an internal model to predict the future system dynamics over a finite prediction horizon and solves a constrained optimization problem with a certain optimization algorithm. MPC uses online optimization, which means the optimization is conducted at every control time step and only the first control action over the prediction horizon is applied to the system. A typical MPC control problem in open channel water management is to maintain a water level distant downstream of a control structure at the end of the canal reach, which will also be the subject of this paper. In the following sections, we discuss the main components of MPC for such a system: internal model and optimization.

2.1. Open channel flow model

In order to control the open channel flow with MPC, the dynamics of the system needs to be properly defined in the internal model of the controller. Open channel flow dynamics is usually described by the Saint-Venant equations, which contain the mass and momentum conservations [12] shown in Eqs. (1) and (2):

$$\frac{\partial A_w}{\partial t} + \frac{\partial Q}{\partial x} = q_l \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial (Q\,v)}{\partial x} + gA_w\frac{\partial \varsigma}{\partial x} + g\frac{Q|Q|}{C_z^2 R A_w} = 0 \tag{2}$$

where A_w is the wetted area [m^2], Q is the flow [m^3/s], q_l is the lateral inflow per unit length [m^3/s/m], v is the average flow velocity [m/s], which equals Q/A_w, ς is the water level above the reference plane [m], C_z is the Chezy coefficient [m$^{1/2}$/s], R is the hydraulic radius [m], which equals A_w/P_f (P_f is the wetted perimeter [m]) and g is the gravity acceleration [m/s^2], Δt is time step [s] and Δx is spatial increment [m].

According to Stelling and Duinmeijer [13], the Saint-Venant equations can be spatially discretized with staggered grids. A semi-implicit scheme is applied to the time integration, where the advection term in the momentum equation is explicitly discretized by a first-order upwind method. The friction term is linearized by using $|Q|$ explicitly. All other terms are implicit. In this way, the Saint-Venant equations are linearized at every time step. Substituting the velocities of step $n + 1$ from the discretized version of Eq. (2) into Eq. (1), the water levels can be calculated with a tridiagonal system, and the velocities are updated with the calculated water levels through the momentum Eq. (2). The detailed discretization of the Saint-Venant equations is included the Appendix.

In general, there is no specific format for the model constraints in MPC. However, in QP-based MPC, the internal model is usually formulated as a linear state-space system. Due to the inter-connection between water levels and velocities, the Saint-Venant equations are approximated by a linear state-space model that is time-varying as shown in Eq. (3):

$$x^{k+1} = A^k x^k + B_u^k u^k + B_d^k d^k \tag{3}$$

where x is the state vector, A is the state matrix, u is the control input vector, B_u is the control input matrix, d is the disturbance vector, B_d is the disturbance matrix and k is the time step index. The detailed formulation of each matrix is included in the Appendix.

2.2. Generic MPC formulation

Typically, an MPC problem in open channel water management solves the minimization of a quadratic objective function, subject to linear or nonlinear model equality constraints and linear inequality constraints on the control inputs. The reason to use a quadratic objective function is to balance both positive and negative variations of states and control inputs, such as water level deviations from the target level and the change of controlled structure flow. The formulation can then be written for a certain control time step k:

$$\begin{gathered}\min_{X^k,U^k} J(X^k,U^k) = \min_{X^k,U^k}\left\{\sum_{j=0}^{n-1}\left[(x^{k+j+1})^T W_x x^{k+j+1}\right] + \sum_{j=0}^{n-1}[(u^{k+j})^T W_u u^{k+j}]\right\}\\ s.t. h_i(X^k,U^k) = 0 \quad i = 1,\ldots,m_e\\ r_i(X^k,U^k) \leqslant 0 \quad i = 1,\ldots,m_i\end{gathered} \tag{4}$$

where J represents a quadratic objective function, $X^k = [x^{k+1}, \ldots, x^{k+n}]^T$ and $U^k = [u^k, \ldots, u^{k+n-1}]^T$ are the states and control inputs over the prediction horizon with a length of n, h_i and r_i are the ith