Advances in Water Resources 45 (2012) 2–12

Contents lists available at SciVerse ScienceDirect

Advances in Water Resources

journal homepage: www.elsevier.com/locate/advwatres

Scaling analysis of the variability of the rain drop size distribution at small scale

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ARTICLE INFO

Article history:
Available online 9 January 2012

Keywords:
Raindrop size distribution
Scaling
Weather radar

ABSTRACT

Like precipitation, the raindrop size distribution (DSD) is strongly variable in space and time. Understanding this variability is important for quantifying and minimizing some of the uncertainties in radar measurements and their interpretation in terms of rain rate. At the typical operational radar pixel scale (i.e., 1×1 km^2), the variability of the DSD is not well documented and understood. A network of 16 identical disdrometers deployed over a 1×1 km^2 area provides an adequate data set to investigate this small-scale variability of the DSD. The single-moment and double-moment DSD scaling approaches are used to analyze the DSD variability for a set of 36 rain events of various types. At fine temporal resolutions, neither the single-moment nor the double-moment normalization capture all the DSD variability, and the scaled DSDs appear different at the point and at the pixel scales. The double-moment normalization can however be used to obtain reliable estimates of the DSD moments at the pixel scale from point measurements, providing a way to upscale DSD moments. At coarser temporal resolutions, the spatial variability within the pixel becomes negligible, and the scaled DSDs are similar at the two spatial scales.

1. Introduction

Given the large number of raindrops in a given volume of rainfall, the rain drop size distribution (DSD hereinafter) is a convenient statistical way to summarize the variety of drop sizes encountered. The DSD reflects the microphysical and dynamic processes at work in the clouds and during the fall of raindrops (see [1] for example). From a remote sensing point of view, DSD is crucial to understand measurements from weather radars, and to convert these measurements into values of rain rate, the variable of interest for many quantitative applications.

Because of the complex interactions between these microphysical processes and turbulence in the atmosphere, DSD (like precipitation in general) is strongly variable in space and time e.g., [2–5]. This variability has an influence on radar measurements and their quantitative interpretation as rain rate estimates [6]. There is however a lack of understanding of the DSD variability at small spatial scales, mainly because of a lack of adequate measurements. Miriovsky et al. [7] have investigated this issue, but because of the limited number of instruments involved and/or because of their different types, the analyses were not conclusive in terms of the quantification of the spatial variability of the DSD at small scales. Recently, Lee et al. [8] have also studied the spatial variability of the DSD using data collected from 4 precipitation occurrence sensor systems (POSS) and an X-band weather radar, and Tokay and Bashor [9] have studied the spatial correlation of some moments of the DSD using data from 3 Joss-Waldvogel disdrometers. These investigations of the spatial structure of (moments) of the DSD were also limited by the small number of instruments.

To tackle this issue, a network of 16 identical optical disdrometers (Parsivel) has been deployed over a typical radar pixel (1×1 km^2) on EPFL campus in Lausanne, Switzerland, in Spring 2009 [10]. The data collected with this network allow to investigate the variability of the DSD at small scales, which is very relevant for radar rain-rate retrieval. The DSD scaling approach proposed by Sempere-Torres et al. [11] and further generalized by Lee et al. [12] has been developed to investigate the variability of the DSD and to take it into account through scaling parameters and scaled DSD functions. It generalizes and unifies other normalization techniques of the DSD e.g., [13,14]. Investigating the link between turbulence and raindrop characteristics, Lovejoy and Schertzer [15] proposed a normalization of the drop mass (and not size) distribution based on two dimensional quantities related to mass and length (the drop mass concentration and the mean drop mass). This alternative normalization approach will not be considered in this study as we focus on the DSD.

In the present work, the scaling approach is used in order to investigate the DSD variability within a radar pixel scale. The main objective is to investigate if the DSD variability can be completely described and quantified by the variability of one or two DSD moments, or if it is related to more complex variabilities of the involved microphysical and dynamic processes. In other words, can the scaling approach be used to upscale the (point) DSD measurements at the radar pixel scale? To answer this question, the uncertainty due to measurement errors must be considered.

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doi:10.1016/j.advwatres.2011.12.016

This article is organized as follows: Section 2 presents the scaling approach, while Section 3 describes the data used in the present analysis. Section 4 explains how the uncertainty due to measurement errors is quantified and taken into account. The main results are detailed in Section 5, and the conclusions are given in Section 6.

2. Methodology

2.1. Single-moment normalization

Sempere-Torres et al. [11] proposed a general scaling formalism based on the normalization of the DSD by a single moment (of the DSD). Sempere-Torres et al. [16] have experimentally investigated the validity of the single-moment normalization approach. This approach has been successfully employed to study the microstructure of extreme precipitation [17], the variability of the DSD in a squall line [4] or the variability of the Z-R relationship for intense Mediterranean rainfall [18].

Let D (mm) be the equivolume diameter of a raindrop, and $N(D)$ ($\mathrm{mm^{-1}\,m^{-3}}$) the DSD. $N(D)dD$ is hence the number of drops per m^3 with a diameter in $[D, D + \mathrm{d}D]$. $N(D)$ can also be written as the ratio between the total concentration of drops N_t ($\mathrm{m^{-3}}$) and a characteristic diameter D_c (mm), multiplied by a probability density function $f\,(-)$ obtained by normalizing the diameter by D_c:

$$N(D) = \frac{N_t}{D_c} f\left(\frac{D}{D_c}\right) \tag{1}$$

Using this notation, the moment of order n can be written as

$$M_n = C_n N_t D_c^n \int_0^\infty \left(\frac{D}{D_c}\right)^n f\left(\frac{D}{D_c}\right) \mathrm{d}\left(\frac{D}{D_c}\right) \tag{2}$$

where C_n is a dimensionless constant for the units. The diameter D is supposed to vary between 0 and ∞. It is assumed that N_t and D_c can be expressed as power laws of a given reference moment of the DSD:

$$N_t = \alpha_{N_t} M_i^{\beta_{N_t}} \tag{3}$$

$$D_c = \alpha_{D_c} M_i^{\beta_{D_c}} \tag{4}$$

where M_i is the reference moment of order i:

$$M_i = C_i \int_0^\infty D^i N(D)\,\mathrm{d}D \tag{5}$$

Injecting Eqs. (3) and (4) in (1) leads to

$$N(D, M_i) = \frac{\alpha_{N_t}}{\alpha_{D_c}} M_i^{\beta_{N_t} - \beta_{D_c}} f\left(\frac{D}{\alpha_{D_c} M_i^{\beta_{D_c}}}\right) \tag{6}$$

which can be written

$$N(D, M_i) = M_i^{\alpha} g(x_1) \tag{7}$$

where $x_1 = D M_i^{-\beta}$ is a normalized diameter ($\mathrm{mm}^{1-i\beta}\,\mathrm{m}^{3\beta}$). The scaling parameters α and β are defined as

$$\alpha = \beta_{N_t} - \beta_{D_c} \tag{8}$$

$$\beta = \beta_{D_c} \tag{9}$$

g is called the single-moment scaled DSD function ($\mathrm{mm}^{-(1+i\alpha)}\,\mathrm{m}^{-3(1-\alpha)}$). From Eq. (7), we have

$$M_n = C_{1,n} M_i^{\gamma_n} \tag{10}$$

with

$$C_{1,n} = \int_0^\infty x_1^n g(x_1)\,\mathrm{d}x_1 \tag{11}$$

$$\gamma_n = \alpha + \beta(n+1) \tag{12}$$

The index 1 in $C_{1,n}$ indicates that the single-moment normalization is considered. From Eq. (12), we see that β can be estimated as the slope of the linear regression of γ_n as a function of the moment order plus 1. Using $n = i$, in Eqs. (11) and (12), we obtain the self-consistency constraints:

$$C_{1,i} = 1 = \int_0^\infty x_1^i g(x_1)\,\mathrm{d}x_1 \tag{13}$$

$$\alpha + \beta(i+1) = 1 \tag{14}$$

According to Eq. (10), any moment of the DSD can be expressed as a power law of the reference moment M_i, with prefactor and exponent being functions of the scaling parameters α and β as well as of the order n of the considered moment.

The rain rate R ($\mathrm{mm\,h^{-1}}$), defined as

$$R = 6\pi 10^{-4} \int_0^\infty D^3 v(D) N(D)\,\mathrm{d}D \tag{15}$$

where $v(D)$ ($\mathrm{m\,s^{-1}}$) is the fall velocity of a drop of diameter D. Assuming $v(D) = 3.778 D^{0.67}$ [19], R corresponds to a moment of order 3.67, and will be used as reference moment for the single-moment normalization, similarly to [4,11]. In this case, x_1 is expressed in $\mathrm{mm}^{1-\beta}\,\mathrm{h}^{\beta}$, and g in $\mathrm{mm}^{-(1+\alpha)}\,\mathrm{h}^{\alpha}\,\mathrm{m}^{-3}$.

As illustrated by Uijlenhoet et al. [17], the scaled DSD function g can be modeled using a gamma function:

$$\hat{g}(x_1) = \kappa_1 x_1^{\mu_1 - 1} e^{-\lambda_1 x_1} \tag{16}$$

Injecting Eq. (16) in the self-consistency constraint Eq. (13) imposes:

$$\kappa_1 = \frac{\lambda_1^{\mu_1 + i}}{C_i \Gamma(\mu_1 + i)} \tag{17}$$

If $i = 3.67$, then $C_i = 6\pi 10^{-4} \times 3.778$.

2.2. Double-moment normalization

Lee et al. [12] have extended the single-moment DSD scaling approach and proposed a double-moment normalization, unifying the different existing DSD normalization techniques e.g., [13,14]. The double-moment normalization has been used to investigate the characteristic microstructure of tropical rainfall [20] as well as the spatial variability of the DSD over a few kilometers [8].

Using Eq. (2) for two reference moments of order i and j, N_t and D_c can be expressed as double power laws of these two moments M_i and M_j:

$$N_t = C_{2,N_t} M_i^{\beta_{N_t,i}} M_j^{\beta_{N_t,j}} \tag{18}$$

$$D_c = C_{2,D_c} M_i^{\beta_{D_c,i}} M_j^{\beta_{D_c,j}} \tag{19}$$

Injecting Eqs. (18) and (19) into Eqs. (1) and (5) leads to:

$$N(D, M_i, M_j) = M_i^{(j+1)/(j-i)} M_j^{(i+1)/(i-j)} h(x_2) \tag{20}$$

where $x_2 = D M_i^{1/(j-i)} M_j^{-1/(j-i)}$ is dimensionless. h is called the double-moment scaled DSD function and is also dimensionless. From Eq. (20), we have

$$M_n = C_{2,n} M_i^{(j-n)/(j-i)} M_j^{(n-i)/(j-i)} \tag{21}$$

with

$$C_{2,n} = \int_0^\infty x_2^n h(x_2)\,\mathrm{d}x_2 \tag{22}$$

The index 2 in $C_{2,n}$ indicates that the double-moment normalization is considered. Using $n = i$ and $n = j$ in Eq. (22), we obtain the self-consistency constraints: