Advances in Water Resources 40 (2012) 1–10

Contents lists available at SciVerse ScienceDirect

Advances in Water Resources

journal homepage: www.elsevier.com/locate/advwatres

Solute transport in partially-saturated deformable porous media: Application to a landfill clay liner

H.J. Zhang [a], D.-S. Jeng [a,*], B.R. Seymour [b], D.A. Barry [c], L. Li [d]

[a] Division of Civil Engineering, University of Dundee, Dundee DD1 4HN, UK
[b] Department of Mathematics, University of British Columbia, Vancouver BC, Canada V6Z 1Z2
[c] Laboratoire de technologie écologique, Faculté del'environnement naturel, Architectural et construit (ENAC), Station 2, Ecole Polytechnique Fédérale de Lausanne (EPFL), 1015 Lausanne, Switzerland
[d] National Centre for Groundwater Research and Training, School of Civil Engineering, The University of Queensland, Queensland, Australia

ARTICLE INFO

Article history:
Received 21 September 2011
Received in revised form 18 January 2012
Accepted 23 January 2012
Available online 8 February 2012

Keywords:
Biot consolidation
Advection–diffusion
Pore water compressibility

ABSTRACT

Based on the one-dimensional Biot consolidation equations, this paper developed an advection–diffusion equation that incorporates saturation, compressibility of the pore fluid and longitudinal dispersivity of the solute transport in an unsaturated, deforming porous medium. A simplified model was proposed for the case of a landfill liner. Numerical results demonstrated that the longitudinal dispersivity and compressibility of the pore fluid can be significant. Furthermore, the degree of soil saturation and loading rate of the waste surcharge affect significantly the contamination advective emission, namely the cumulative contaminant mass outflow per unit area from compacted clay liner (CCL) due to advective flow.

1. Introduction

Various environmental situations are typically investigated using solutions of the solute transport equations considering the porous medium to be rigid (e.g., [1–5]). In such cases, no volume change occurs during the transport process and therefore the advection is determined solely by the hydraulic gradient. However, porous medium deformations can lead to unsteady advective flow. Some examples include solute transport through a clay liner during waste-filling operations, dredged contaminated sediment after placement in a confined disposal facility, consolidation of contaminated sediments due to overburden of capping material, and solute transport in cartilage under mechanical load (e.g., [6–8]). In these cases, the deformation and solute transport processes occur simultaneously and coupled effects should be considered.

Modeling of contaminant transport through deformable porous media has received attention during the last two decades. Potter et al. [9] presented a model for dissolved phase advection–dispersion transport using Terzaghi consolidation theory. Smith [6] derived a one-dimensional theory of contaminant migration based on a small strain analysis of a consolidating soil. The equations were recast in a material coordinate system for problems involving large deformation or a moving boundary. Peters and Smith [10] extended the previous model of Smith [6] for transient solute transport within a deformable porous medium for both small and large deformations. Moo-Young et al. [11] presented experimental results of contaminant transport in soil specimens undergoing consolidation induced by a centrifuge; consolidation was observed to accelerate solute migration. With the coupled Terzaghi consolidation and ADE equation (advection–diffusion equation), Alshawabkeh et al. [12] calculated the contaminant mass flux that was enhanced by the capping load-induced sediment consolidation. They concluded that advection caused by consolidation will accelerate the breakthrough of the contaminant through the cap [12]. Arega and Hayter [7] used a one-dimensional large strain consolidation and contaminant transport model to simulate capping consolidating contaminated sediment based on reduced coordinates.

In addition to the approaches for small strain, Lewis [13] generalized the finite strain consolidation and solute transport model of Peters and Smith [10] by incorporating self-weight in the consolidation process, and included more general constitutive functions for consolidation and transport coefficients. Fox and co-workers [14–16] adopted a piecewise linear approach to handle coupled one-dimensional large strain consolidation and two-dimensional solute transport in a confined disposal facility for dredged contaminated sediments.

In real environments, unsaturated porous media are common [17–19]. For example, marine sediments are often unsaturated due to gas produced in biochemical processes. Another case is where the groundwater table is located some distance below a landfill geomembrane, in which case the soil beneath the landfill will be partially saturated [17].

* Corresponding author.
E-mail address: d.jeng@dundee.ac.uk (D.-S. Jeng).

0309-1708/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.advwatres.2012.01.007

List of symbols

A_i	coefficients in dimensionless equations
c^*	non-dimensional mass concentration of the solute in the fluid phase
c_0	the reference solute mass concentration, ML^{-3}
c_f	concentration of the solute in the fluid phase, ML^{-3}
c_s	concentration of the solute in the solid phase, ML^{-3}
c_v	coefficient of consolidation, L^2T^{-1}
D	hydrodynamic dispersion coefficient, L^2T^{-1}
D_G	mass transfer coefficient of geomembrane, L^2T^{-1}
D_m	effective molecular diffusion coefficient, L^2T^{-1}
F	function to relate S and c_f
G	shear modulus of soil, $ML^{-1}T^{-2}$
g	gravity acceleration, LT^{-2}
h	thickness of geomembrane, L
K	hydraulic conductivity, LT^{-1}
K_d	contaminant partitioning coefficient, L^3M^{-1}
K_{w0}	pore water bulk modulus, $ML^{-1}T^{-2}$
L	thickness of CCL, L
l_c	characteristic unit for length, L
n	current soil porosity
n^0	initial soil porosity
P_a	gauge air pressure, $ML^{-1}T^{-2}$
P_0	atmosphere air pressure, $ML^{-1}T^{-2}$
p^*	non-dimensional excess pore water pressure
p_c	characteristic unit of excess pore pressure, $ML^{-1}T^{-2}$
p^e	excess pore pressure, $ML^{-1}T^{-2}$
r_h	volumetric fraction of dissolved air
S	mass of contaminant sorbed onto the solid phase per unit mass of solid phase
S_r	degree of saturation
T_v	consolidation time factor in Terzaghi consolidation theory, T
t	time, T
t^*	non-dimensional time
t_c	characteristic unit for time, T
u	soil displacement, L
u^*	non-dimensional soil displacement
u_c	characteristic unit for soil displacement, L
v_f	average fluid velocity, LT^{-1}
v_s	solid velocity, LT^{-1}
z	vertical coordinate, L
z^*	non-dimensional vertical coordinate

Greek symbols

ρ_w	density of pore water, ML^{-3}
ρ_s	density of soil gain, ML^{-3}
β	compressibility of pore water, LT^2M^{-1}
v	Poisson's ratio
α_L	longitudinal dispersion, L

In this study, the case of a landfill liner is considered, for which a one-dimensional Biot consolidation equation is used to describe flow in an unsaturated porous medium incorporating the self-weight of the liner. The situation considered is that of compressible pore water at a fixed saturation. The ADE that is typically used to describe solute transport through a rigid porous medium [1] is modified to include partial saturation, CPW (compressible pore water), SVP (spatial variation of porosity) and longitudinal dispersivity. The equations are non-dimensionalized, identifying nine important parameters. The importance of these parameters is discussed for a range of physical conditions. A hypothetical engineered landfill liner is used as an illustrative example, demonstrating the influence of partial saturation and the loading process on contaminant migration.

2. Theoretical formulation

2.1. Consolidation equation

Here we state the basic equations linking flow velocity with excess pore pressure. The one-dimensional unsaturated fluid storage (Appendix A) and Biot equations [20] are, respectively,

$$S_r n\beta\frac{\partial p^e}{\partial t}+S_r\frac{\partial^2 u}{\partial t\,\partial z}=\frac{1}{\rho_w g}\frac{\partial}{\partial z}\left(K\frac{\partial p^e}{\partial z}\right), \tag{1}$$

$$G\frac{2(1-v)}{(1-2v)}\frac{\partial^2 u}{\partial z^2}+(1-n^0)(\rho_s-\rho_w)g\frac{\partial u}{\partial z}=\frac{\partial p^e}{\partial z}, \tag{2}$$

where p^e is excess pore pressure, u is soil displacement. S_r, n, n^0, K, G and v represent degree of saturation, current porosity, initial porosity, hydraulic conductivity, shear modulus and Poisson's ratio, respectively; ρ_w, ρ_s are the density of pore water and solid materials, respectively. Note that the compressive effective normal stress is negative.

In this study, density of both components of soil are independent of the dilute solute concentration [21]. When the sorption occurs, the mass of a unit volume of solid grains (i.e., density) ρ_s becomes $\rho_s(1 + K_d c_f)$. Using the clay liner as an example, the measured VOC concentration in the landfill leachate ranges from 10 to 10^4 µg/l [21]. Lewis et al. [22] adopted the distribution coefficient K_d = 1 mg/l, leading to the change of the density of solid due to sorption is less than 0.001%, which is negligible. Consequently, it is reasonable to assume that ρ_s is independent of the solute mass concentration. Therefore, the assumption of volume-preserving deformation of the solid phase embedded in derivation (Appendix A) can be ensured, i.e., $\nabla \bullet \vec{v}_s = 0$ [23].

The compressibility of pore fluid in clay, β, depends on the degree of saturation S_r, the amount of dissolved air in pore water and absolute air pressure. It can be estimated by [24]

$$\beta=\frac{S_r}{K_{w0}}+\frac{1-S_r+r_hS_r}{P_a+P_0}, \tag{3}$$

where K_{w0} is the pore water bulk modulus, r_h denotes volumetric fraction of dissolved air within pore water, P_a denotes gauge air pressure and P_0 represents the atmosphere pressure. In the high saturation limit, when r_h = 0.02, S_r = 0.8–1.0 and β falls in the range of 2×10^{-6}–2×10^{-7} Pa^{-1}.

2.2. Solute transport equation

Following [10], the solute transport equation in a one-dimensional deforming porous medium is

$$\frac{\partial(nS_rc_f)}{\partial t}+\frac{\partial[(1-n)c_s]}{\partial t}=-\frac{\partial}{\partial z}\left[nS_r\left(-D\frac{\partial c_f}{\partial z}+v_fc_f\right)+(1-n)v_sc_s\right], \tag{4}$$

where c_f and c_s are the concentration of the solute in the fluid and solid phase, respectively; D, which represents the hydrodynamic dispersion coefficient, is the sum of the effective molecular diffusion, D_m, and mechanical dispersion, $\alpha_L(v_f - v_s)$, where v_f denotes the average fluid velocity and v_s is the velocity of the solid. Here,