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Multilayer shallow water flow using lattice Boltzmann method with high performance computing

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ABSTRACT

A multilayer lattice Boltzmann (LB) model is introduced to solve three-dimensional wind-driven shallow water flow problems. The multilayer LB model avoids the expensive Navier–Stokes equations and obtains stratified horizontal flow velocities as vertical velocities are relatively small and the flow is still within the shallow water regime. A single relaxation time BGK method is used to solve each layer coupled by the vertical viscosity forcing term. To increase solution stability, an implicit step is suggested to obtain flow velocities. The main advantage of using the LBM is that after selecting appropriate equilibrium distribution functions, the LB algorithm is only slightly modified for each layer and retains all the simplicities of the LBM within the high performance computing (HPC) environment. The performance of the parallel LB model for the multilayer shallow water equations is investigated on CPU-based HPC environments using OpenMP. We found that the explicit loop control with cache optimization in LBM gives better performance on execution time, speedup and efficiency than the implicit loop control as the number of processors increases. Numerical examples are presented to verify the multilayer LB model against analytical solutions. We demonstrate the model's capability of calculating lateral and vertical distributions of velocities for wind-driven circulation over non-uniform bathymetry.

1. Introduction

In the last decade, the lattice Boltzmann (LB) method or LBM has become a powerful numerical method for simulating fluid flows [1]. The method is based on statistical physics and simulates fluid flow by tracking the evolution of distribution functions of fluid particles in discrete phase (velocity) space. The essential approach in the LB method lies in the recovery of macroscopic fluid flows from the microscopic flow behavior of the particle movement. The actual particles are not tracked, but their collective behavior is tracked through the mesoscopic evolution of particle distributions. The basic idea is to replace the nonlinear differential equations of macroscopic fluid dynamics by a simplified description modeled on the kinetic theory of gases [2]. The benefit of this description is that the LBM does not involve the solution of a global system of equations, but instead has locality, which makes it very suitable for parallel computing. Recently, this has become an important feature for numerical methods as high performance computing (HPC) systems are currently being designed to solve large-scale engineering problems.

The LBM was first developed to solve the equations of hydrodynamics governed by the Navier–Stokes equations based on the kinetic theory of gases described by the Boltzmann equation [3]. The method has been shown to be effective for simulating flows in complicated geometries on parallel computer architectures [4]. Furthermore, the method has become an alternative to other numerical methods, e.g. finite difference, finite element, and finite volume methods, in computational fluid dynamics. Due to its attractive features, recently the LBM has found a wide range of applications including wind-driven ocean circulation [5,6], discontinuous flows with shocks [7,8], and tidal flows on complex geometries with irregular bathymetry [9].

Even with increasing interest in using the LBM to solve the shallow water equations, its application is limited to two-dimensional planar problems [10–12]. When stratified horizontal velocities in depth are of interest, solving the depth-averaged shallow water equations is not sufficient. However, a full three-dimensional model of the Navier–Stokes equations is computationally expensive and does not yield much more information as the vertical velocities are relatively small and the flow is still within the shallow water flow regime. In order to take advantage of the shallow water equations while avoiding the drawbacks of the depth-averaged models, a multilayer system [13] was adopted in this study. The multilayer shallow water equations have been solved

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by finite difference, finite element, and finite volume methods. Abgrall and Karni [14] developed a relaxation scheme to solve bi-fluid shallow water problems. The solution procedure has some similarities including propagation and relaxation; however, the present work deals with a single fluid and the solution procedure of the LBM is performed in the kinetic model. Audusse et al. [15] extended previous works [16,17] to model dam breaks using a finite volume solver. Their finite volume method is based on a continuous kinetic model to calculate the fluxes between cells. In this perspective, the LBM can be viewed as a discrete kinetic model. A detailed comparison of the LBM and continuous kinetic schemes can be found in [18]. Continuous kinetic schemes such as the gas kinetic scheme were found to be more memory efficient while the discrete kinetic LBM is computationally more efficient based on being three times faster. To the authors' best knowledge, the LBM has not been applied to the multilayer shallow water equations with HPC. Due the simplicity of the multilayer LB model, it can be an efficient numerical method for more complex flow and transport problems and can be easily extended to include, for example, turbulence models and quadratic friction laws while retaining the parallel advantages.

The objective of this study is to develop a parallelized LB model to solve the multilayer shallow water equations for three-dimensional shallow water flow problems on irregular boundary geometry and bathymetry. The LB method using a single relaxation time Bhatnagar–Gross–Krook (BGK) collision operator [19] (LBGK) was adopted to solve each layer coupled by the vertical viscosity forcing term. To increase solution stability, an implicit step is suggested to obtain flow velocities. The main advantage of using the LBM is that after selecting appropriate equilibrium distribution functions, the LB algorithm is only slightly modified for each layer and retains all the simplicities of the LBM within the HPC environment. We focus on the application of the multilayer shallow water equations to the problems of wind-driven circulation. We implement and investigate the parallel performance of the multilayer LB method on a shared memory HPC system, an AIX v5.3 constellation from IBM with 1.9 GHz IBM POWER5+ processors housed in the Center for Computation & Technology (CCT), Louisiana State University. We also demonstrate smaller case studies on a single workstation with a 3.0 GHz Intel® Core™2 Extreme quad core to illustrate parallel speedup on a single workstation. In this study, our LB code was written in Fortran 90 and parallelized with OpenMP using flow domain decomposition and cache optimization.

In this study, we use a parallel decomposition based on dividing the flow domain along the lateral flow direction, not based on the number of layers. A parallel decomposition based on layers would be challenging since the number of layers is always smaller than the number of nodes in the longitudinal or lateral directions. Furthermore, this decomposition would be difficult to balance workloads and require more communication between processors during the LB and implicit step. On the other hand, the decomposition along the lateral flow direction retains the inherent parallelism of the LBM by limiting the communication between processors. It also ensures that the vertical coupling between layers remains local to each processor, which is important for the implicit step.

The paper is organized as follows. In Section 2, the multilayer shallow water equations are introduced. Section 3 introduces the multilayer LB model and recovers the macroscopic equations up to second order accuracy through the Chapman–Enskog expansion. Section 4 discusses the parallel computing implementation of the multilayer LB model on shared memory systems. Section 5 presents numerical results to verify the model results against analytical solutions and demonstrate its ability to model stratified horizontal velocities. Section 6 concludes the study.

2. Multilayer shallow water equations

A multilayer model is considered by converting a three-dimensional shallow water flow problem into a number of coupled two-dimensional shallow water flow problems in layers as shown in Fig. 1. Based on the multilayer Saint–Venant system [13], the governing equations are similar to the traditional shallow water equations with additional terms for transferring momentum between the layers:

$$\frac{\partial h^{(\ell)}}{\partial t} + \frac{\partial\left(h^{(\ell)}u_i^{(\ell)}\right)}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial\left(h^{(\ell)}u_i^{(\ell)}\right)}{\partial t} + \frac{\partial\left(h^{(\ell)}u_i^{(\ell)}u_j^{(\ell)}\right)}{\partial x_j} + \frac{\partial}{\partial x_i}\left(\frac{1}{2}gh^{(\ell)}\sum_{m=1}^{M}h^{(m)}\right) = \nu\left[\frac{\partial^2\left(h^{(\ell)}u_i^{(\ell)}\right)}{\partial x_j \partial x_j}\right] + F_i^{(\ell)}, \quad \ell = 1,2,\ldots,M \tag{2}$$

where $h^{(\ell)}$ is the local water height in layer ℓ, $u_i^{(\ell)}$ is the local velocity component in the i direction in layer ℓ, $F_i^{(\ell)}$ is the external force acting on layer ℓ, g is the gravitational acceleration, ν is the kinematic viscosity, x_i is the Cartesian coordinate, and t is time. M is the total number of layers. The external force consists of the wind-driven forcing term $\left(F_{Wi}^{(\ell)}\right)$ (only for the top layer), the bed slope forcing term $\left(F_{Pi}^{(\ell)}\right)$, the vertical kinematic eddy viscosity term $\left(F_{\mu i}^{(\ell)}\right)$, the non-conservative pressure source term $\left(F_{NCi}^{(\ell)}\right)$ [13,15–17], and the Coriolis forcing term $\left(F_{Ci}^{(\ell)}\right)$ as follows

$$F_i^{(\ell)} = F_{Wi}^{(\ell)} + F_{Pi}^{(\ell)} + F_{\mu i}^{(\ell)} + F_{NCi}^{(\ell)} + F_{Ci}^{(\ell)} \tag{3}$$

$$F_{Wi}^{\ell} = \delta_{M\ell}\frac{\tau_{iz}^{W}}{\rho} = \delta_{M\ell}\frac{\rho_a}{\rho}C_W U_{Wi} W_s \tag{4}$$

$$F_{Pi}^{(\ell)} = -gh^{(\ell)}\frac{\partial z_b}{\partial x_i} \tag{5}$$

$$F_{\mu i}^{(\ell)} = -\kappa\delta_{1\ell}u_i^{(\ell)} + 2\mu(1-\delta_{M\ell})\frac{u_i^{(\ell+1)} - u_i^{(\ell)}}{h^{(\ell+1)} + h^{(\ell)}} - 2\mu(1-\delta_{1\ell})\frac{u_i^{(\ell)} - u_i^{(\ell-1)}}{h^{(\ell)} + h^{(\ell-1)}} \tag{6}$$

$$F_{NCi}^{(\ell)} = -\frac{gH^2}{2}\frac{\partial}{\partial x_i}\left(\frac{h^{(\ell)}}{H}\right) \tag{7}$$

$$F_{Ci}^{(\ell)} = \begin{cases} f_c h^{(\ell)} u_y, & i = x \\ -f_c h^{(\ell)} u_x, & i = y \end{cases} \tag{8}$$

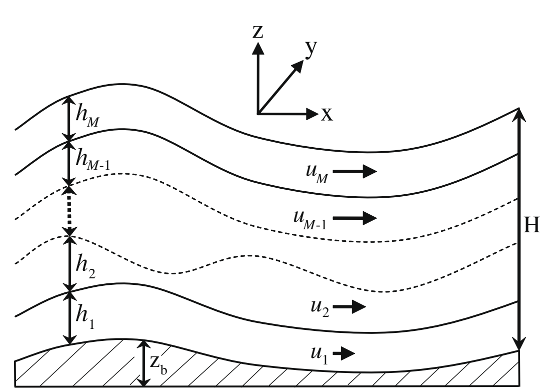

Fig. 1. Multilayer shallow water discretization.