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Simulation of density-driven flow in fractured porous media

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ABSTRACT

We study density-driven flow in a fractured porous medium in which the fractures are represented as manifolds of reduced dimensionality. Fractures are assumed to be thin regions of space filled with a porous material whose properties differ from those of the porous medium enclosing them. The interfaces separating the fractures from the embedding medium are assumed to be ideal. We consider two approaches: (i) the fractures have the same dimension, d, as the embedding medium and are said to be d-dimensional; (ii) the fractures are considered as $(d-1)$-dimensional manifolds, and the equations of density-driven flow are found by averaging the d-dimensional laws over the fracture width. We show that the second approach is a valid alternative to the first one. For this purpose, we perform numerical experiments using finite-volume discretization for both approaches. The results obtained by the two methods are in good agreement with each other.

1. Introduction

The study of fractured porous media is an important and challenging problem in hydrogeology. One of the difficulties is that mathematical models have to account for heterogeneity introduced by fractures in hydrogeological media. Heterogeneity may strongly influence the physical processes taking place in these media. Moreover, the thickness of the fractures, which is usually negligible in comparison with the size of the whole domain, and the complicated geometry of fracture networks reduce essentially the efficiency of numerical methods. In order to overcome these difficulties, fractures are sometimes considered as objects of reduced dimensionality (surfaces in three dimensions), and the field equations are averaged along the fracture width (cf., for example, [1,4,5,38,49]).

From a descriptive point of view, a fracture is a portion $\mathscr{F}$ of a region of observation, $\Omega \subset \mathbb{R}^d$ (where $d = 2$ or $d = 3$), characterized by a shape such that one of its geometric dimensions is much smaller than the other ones [10]. In Ω, there can be either isolated fractures or networks of fractures. Fracture networks usually provide a resistance to the motion of groundwater lower than that of unfractured rock, and may thus lead to the movement of a comparatively large amount of water and the substances dissolved in it. A quantification of flow and transport in fractured rocks was undertaken by Neumann [42]. Because of its influence on the environment (e.g., pollution of aquifers), and its connection with industrial problems (e.g., modelling of fractured reservoirs [10]), the flow in fractured porous media has received particular attention [41,37,46,22,24,45]. Mathematical and numerical models have been developed in order to predict fluid flow and contaminant transport in highly heterogeneous domains, the heterogeneity being given by the abrupt change of permeability when passing from the fracture to the surrounding domain, and vice versa.

Together with heterogeneity, the spatial distribution of the fractures in a given network may also influence other flow properties. For example, even though the permeability of the medium embedding the fractures is isotropic, the effective permeability of the equivalent system, made of the network and the embedding medium, may be anisotropic.

The evaluation of the effective flow properties of heterogeneous media is based upon upscaling methods such as, for example, volume-averaging [13,27,50], asymptotic expansions [12], stochastic modelling, and coarse graining and Renormalization Group Theory [2]. In the context of volume-averaging, the method known as average-along-the-vertical [8] is applied to thin flow regions. The description of flow and transport provided by this method is based on averaging the equations defined in the flow region along its width. In our paper, the thin flow region is represented by a fracture.

Among the modelling approaches for fractured porous media we can classify two approaches: the near-field and far-field. The first one considers a relatively small domain with a small number of well-defined fractures whose location and shape is known. The second approach is based on the concept of overlapping continua [10], which are identified by the fluid in the fractures and the fluid in the embedding medium. If the region of investigation is large with respect to the size of the fractures but not big enough to allow

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for the introduction of the overlapping continua, then the near-field approach is used. The model presented in our paper belongs to this case. We assume that the fractures are filled by a porous medium whose permeability is bigger than the permeability of the medium enclosing them. The fractures and the embedding medium preserve their identity, and their mutual interaction is studied by means of interface balance laws. Therefore, in the resulting picture, the overall medium is heterogeneous with respect to permeability, and the more permeable medium is inside the fractures.

In our model, a fracture has the following two properties: (a) its thickness is much smaller than the smallest characteristic length scale of the embedding medium, and (b) it is filled by a porous medium whose porosity and permeability differ from those of the embedding medium. The fractures are thus distinguishable from the medium. For a given fracture, we assume $K_f \gg K_m$, where K_f and K_m are the permeabilities of the fractures and medium, respectively. Our study considers the fractures and the embedding medium at the same scale of observation. Since we model density-driven flow, we need to describe flow and transport in the whole region of observation. In order to do that, we use the same equations both in the fracture and in the embedding medium. Flow in the fracture is such that Darcy's law is still applicable. In order to account for the geometric properties of the fracture, we adopt Bear's procedure [8] and average the equations of density-driven flow over the fracture width. After averaging, the fractures remain distinguishable from the embedding medium but are regarded as objects of reduced dimensionality.

In our numerical verifications, the shape of the fractures is accounted for by the following two different approaches. In the first one, the fractures have the same geometric dimension as the embedding medium and are thus said to be d-dimensional, with $d = 2$ in 2D and $d = 3$ in 3D. In the second approach, the fractures are considered as $(d-1)$-dimensional manifolds, and the equations of density-driven flow are found by averaging the d-dimensional ones over the fracture width. The first approach is well-established, more general, but computationally more expensive. The second approach, instead, requires some working hypotheses but is computationally cheaper. The scope of the paper is to investigate the second approach and to show that it is a valid alternative to the first one for solving flow and transport in a fractured porous medium. In order to prove this, we perform numerical experiments using finite-volume discretization for both the d- and the $(d-1)$-dimensional resolution of the fractures, and we show that the results obtained by the two methods are in good agreement with each other for sufficiently small fracture widths.

The equations obtained within the $(d-1)$-dimensional model (see Section 2) are similar to the equations determined by Cermelli et al. [15], in the study of transport relations for surface integrals defined over evolving surfaces. In [15], the authors elaborate two-dimensional transport models, in which the effect of curvature is accounted for.

In the context of density-driven flow in fractured porous media, our paper presents the following methodological and computational results: (i) the equations modelling flow and transport in the fractures are independent on the equations used in the embedding medium, and the unknown functions in the fractures are not assumed to be equal to the unknown functions in the embedding medium; (ii) the use of the concept of "excess mass" (Murdoch, 1995) in the formulation of the $(d-1)$-dimensional representation of fractures; (iii) the development of numerical schemes for solving the equations of density-driven flow as formulated in the $(d-1)$-dimensional model; (iv) the verification of the proposed $(d-1)$-dimensional model through numerical experiments conducted by using the d-dimensional formulation; (v) a small discussion about the production of vortices in the fractures for some particular fracture geometries and locations of the fracture in the region of observation. The concept of "excess mass" was used in [30] for modelling interfaces, and in the works by Murdoch and Soliman [40] and Murdoch [39], where it was indicated as one of the most important aspects of modelling transport on lower-dimensional domains embedded in three-dimensional regions. Excess quantities are defined only for extensive quantities, and should be taken into account when the real, three-dimensional interface between two regions is replaced with an equivalent domain of lower dimensionality. This should be done, for example, in order not to "gain" an unphysical mass after ideally "shrinking" the fracture to its mean plane $\mathscr{S}$ (see [39] for details).

The paper is organized as follows. In Section 2, we propose the model of the density-driven flow in the fractured media with the low-dimensional representation of the fractures. In Section 3, we present the results of the numerical experiments with the proposed model and compare them with the results obtained by the simulations with the full-dimensional fractures. The finite-volume discretization of the model and the numerical methods used in our simulations are described in Appendix A.

2. Model

We develop our model using Hybrid Mixture Theory [11], where a porous medium is macroscopically modelled as a mixture of solids and fluids, that co-exist in a given region of space $\Omega \subset \mathbb{R}^d$ ($d = 2$ or $d = 3$). In many hydrogeological applications, it is assumed that the mixture consists of a single solid-phase, for example, the porous rocky matrix of a soil, and either a multi-phase or a single-phase fluid. The fluid-phase comprises constituents which undergo physico-chemical processes such as advection, diffusion, chemical reactions, and exchange with the solid-phase.

For our purposes, we restrict our model to the case in which a two-constituent fluid experiences single-phase flow through the porous matrix of the solid-phase. The two constituents of the fluid-phase are assumed to be water and brine, the latter being a chemical compound consisting of salts and water. In the presence of exchange processes between the fluid- and the solid-phase, each of these phases should be regarded as a mixture in which each phase is composed of the same constituents [11]. In the absence of such an assumption it is possible to assume that the fluid- and the solid-phase are a two- and single-constituent system, respectively.

Under the hypothesis that the whole mixture is subject to the saturation condition, the porosity ϕ coincides with the volume fraction of the fluid-phase. The volume fraction of the solid-phase is thus given by $(1-\phi)$.

2.1. Governing equations

In our model, a fracture $\mathscr{F}$ is a region occupied by a porous medium whose permeability is bigger than the permeability of the medium $\mathscr{M}$ in which it is embedded. We assume that the same flow and transport processes occur both in the fracture and in embedding medium. The regions $\mathscr{F}$ and $\mathscr{M}$ interact through exchange processes. To be consistent with the macroscopic continuum description, the partial differential equations governing density-driven flow are obtained by means of the balance laws of mass, momentum, and energy, and the Second Principle of Thermodynamics. These laws have to be written for each constituent of the fluid-phase (i.e., water and brine), and for the solid-phase. However, suitable hypotheses allow for a considerable reduction of the number of equations to be solved. We assume that the fracture and the surrounding porous medium satisfy the following requirements: (a) they are subject to a uniform temperature field;