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Fully coupled mathematical modeling of turbidity currents over erodible bed

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ABSTRACT

Turbidity currents may feature active sediment transport and rapid bed deformation, such as those responsible for the erosion of many submarine canyons. Yet previous mathematical models are built upon simplified governing equations and involve steady flow and weak sediment transport assumptions, which are not in complete accordance with rigorous conservation laws. It so far remains unknown if these could have considerable impacts on the evolution of turbidity currents. Here a fully coupled modeling study is presented to gain new insights into the evolution of turbidity currents. The recent analysis of the multiple time scales of subaerial sediment-laden flows over erodible bed [Cao Z, Li Y, Yue Z. Multiple time scales of alluvial rivers carrying suspended sediment and their implications for mathematical modeling. Adv Water Resour 2007;30(4):715–29] is extended to subaqueous turbidity currents to complement the fully coupled modeling. Results from numerical simulations show the ability of the present coupled model to reproduce self-accelerating turbidity currents. Comparison among the fully and partially coupled and decoupled models along with the analysis of the relative time scale of bed deformation explicitly demonstrate that fully coupled modeling is essential for refined resolution of those turbidity currents featuring active sediment transport and rapid bed deformation, and existing models based on simplified conservation laws need to be reformulated.

1. Introduction

Turbidity currents are driven by the buoyancy force arising from the bulk density excess due to the presence of suspended sediment, and occur in numerous man-made and natural situations. In reservoirs and lakes, turbidity currents are important to the management of siltation and water quality (e.g., [38,42,10,30]). Turbidity currents in oceans are known to be a creator of submarine canyons and fans and have attracted much interest (e.g., [22,32,4,14,39,33]).

Over the recent decades, there have been a number of investigations, including laboratory experiments, field observations and mathematical modeling of turbidity currents because of their profound impacts on inland waters and oceans. Previous laboratory experiments of turbidity currents include Garcia and Parker [16,17], Bonnecaze et al. [3], Hallworth and Huppert [18], Yu et al. [42], Alexander and Mulder [1], Baas et al. [2] and Oehy and Schleiss [30]. However, these may not be able to fully reveal the mechanism of turbidity currents as constrained by the comparatively small spatial scales that can be realistically accommodated in laboratories [27–29]. Field observations (e.g., [22,24,9]) are scarce because of the need to operate underwater, the substantial equipment requirements and the destructive effects that turbidity currents could have on underwater structures [22,15]. Due to the aforementioned issues with laboratory experiments and field observations, mathematical modeling seems to be an attractive alternative for the investigation of turbidity currents.

Mathematical modeling of turbidity currents has been set under distinct frameworks, ranging from models based on classical fluid dynamics principles to new methods that have emerged from alternative computational paradigms such as cellular automata. Cellular automata models have recently been employed to simulate the evolution of turbidity currents [35–37]. In this category of models, the dynamical system of turbidity currents is subdivided into elementary parts, and simulated by updating each elementary part. Yet, the development of cellular automata models for turbidity currents is still in its infancy. Mathematical models based on classical fluid dynamics principles have received great attention in the last several decades, including full three dimensional (3D), vertical two dimensional (2D) and depth-averaged models. Full 3D models (e.g., [10,9,21,23,20,30]) and vertical 2D models [13] normally incorporate a turbulence closure model and can provide detailed flow structure information along the current depth. However, their applications to turbidity currents with active sediment transport and rapid bed deformation are largely hindered because of the movable boundaries (including the free surface and mobile-bed), i.e., they are approximately applicable to cases with steady (or slightly unsteady) flow, mild bed deformation or fixed bed situations. Depth-averaged models have been em-

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Nomenclature

S_b	canyon bed slope	ν	kinematic viscosity of clear-water
d	sediment particle diameter	$\mathbf{U}$	conservative variable vector defined in Eq. (19a)
v_s	sediment settling velocity	$\mathbf{F}$	flux vector defined in Eq. (19b)
p	sediment porosity	$\mathbf{S}$	source vector defined in Eq. (19c)
ρ_w, ρ_s	densities of water and sediment, respectively	$\mathbf{Q}$	primitive variable vector
ρ	density of water–sediment mixture	q	one of the primitive variable vector
ρ_0	density of saturated bed	α, η, β	coefficients
R	submerged specific gravity of sediment	I_i	computing cell
t	time	L, R	superscripts representing the side of computing cell
x	streamwise coordinate	$\bar{\Delta}_i$	limited slope
h	turbidity current thickness	$\mathbf{W}$	primitive variable vector defined in Eq. (34a)
u	layer-averaged velocity	$\mathbf{A}$	matrix defined in Eq. (34b)
c	layer-averaged volumetric sediment concentration	$\mathbf{R}$	vector defined in Eq. (34c)
z	bed elevation	Cr	Courant number
k	mean turbulent energy	$\lambda'_{1,2,3}$	three eigenvalues of the Jacobian matrix $\partial\mathbf{F}/\partial\mathbf{U}$
g	gravitational acceleration	Δt	time step
u_*	bed shear velocity	Δx	spatial step
c_D	bed drag coefficient	Δz	bed scour depth
c_{D^*}	bed drag coefficient at equilibrium conditions	i	spatial node index
r_w	ratio of upper-interface resistance to bed resistance	m	time step index
E, D	sediment entrainment and deposition fluxes	Fr_0	the Froude number at the inlet boundary
e_w	water entrainment coefficient	$\lambda_{1,2,3,4}$	celerities
Ri	Richardson number	T_b	time scale of bed deformation
ε_0	dissipation rate	T_h	time scale of turbidity current thickness
c_b	near bed sediment concentration	T_b/T_h	relative time scale of bed deformation due to turbidity currents
E_s	erosion coefficient		
Z	parameter determining the value of E_s	TC_t	constitute contribution of temporal change
Z_c	value of Z needed for the onset of significant suspension	TC_x	constitute contribution of spatial change
Z_m	value of Z above which E abruptly ceases to increase	R_{tx}	contribution ratio of temporal change to spatial change

ployed reasonably widely to simulate turbidity currents, either decoupled or partially coupled. In decoupled models, bed deformation is not considered at all (e.g., [14,33,6]). In partially coupled models, bed deformation is taken into account in the mass conservation equation of bed material (e.g., [11,12,43,5,34]), yet its feedback impacts on the evolution of turbidity currents are partly neglected. This is because the mass and momentum transfer respectively in the continuity and momentum equations for the water–sediment mixture are exclusively ignored without justification, which arise from sediment exchange with the bed. Pantin [31] proposes a model with full consideration of the mass and momentum transfers due to sediment exchange with the bed; however, bed deformation is not considered at all. Strictly speaking, existing depth-averaged models are built upon simplified governing equations, and not in complete accordance with the rigorous conservation laws. Thus they are only approximately suitable for turbidity currents with weak sediment transport and mild bed deformation or fixed bed situations. Actually, there exists in nature a hierarchy of turbidity currents assuming active sediment transport and rapid bed deformation. For instance, submarine turbidity currents can attain surprisingly swift velocities, as high as 20 m/s (e.g., [19,26,22]). The most telling case is the self-accelerating turbidity currents (e.g., [32,14,33]). It so far remains unknown if simplification of the governing equations could have considerable impacts on the spatial and temporal evolution of turbidity currents. Naturally, the need is evident for physically enhanced models applicable to turbidity currents featuring active sediment transport and rapid bed deformation.

This study aims to investigate fully coupled modeling of turbidity currents with active sediment transport and rapid bed deformation, while still being applicable to situations with steady (or slightly unsteady) and weak sediment transport and mild bed deformation. A fully coupled model is presented, of which the governing equations are cast into a conservative hyperbolic system and numerically solved using the Total-Variation-Diminishing (TVD) version of the second-order Weighted Average Flux (WAF) method along with the Slope Limiter Centered (SLIC) approximate Riemann solver and the MINBEE limiter [40]. The recent analysis of the multiple time scales of subaerial sediment-laden flows over erodible bed by Cao et al. [8] is extended to subaqueous turbidity currents. Particular attention is paid to the feedback impacts of bed deformation on the evolution of turbidity currents.

2. Mathematical formulations

2.1. Governing equations

Consider longitudinally one-dimensional and layer-averaged formulation of turbidity currents over an erodible bed that is composed of uniform and non-cohesive sediment with particle diameter d. Pantin [31] proposes a model with full consideration of the mass and momentum transfers due to sediment exchange with the bed, including the mass and momentum conservation for the water–sediment mixture flow, and the mass conservation of sediment; however, bed deformation is not considered at all. Based on the work of Pantin [31], the mass conservation of bed material is herewith incorporated to reflect the potential bed deformation. Therefore, the complete governing equations read

$$\frac{\partial \rho h}{\partial t} + \frac{\partial \rho h u}{\partial x} = \rho_w e_w u - \rho_0 \frac{\partial z}{\partial t} \tag{1}$$

$$\frac{\partial \rho h u}{\partial t} + \frac{\partial \rho h u^2}{\partial x} + \frac{\partial}{\partial x}\left(\frac{(\rho-\rho_w) g h^2}{2}\right) = -(\rho-\rho_w) g h \frac{\partial z}{\partial x} - \rho u_*^2 (1 + r_w) \tag{2}$$