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Linear and angular momentum conservation in hydraulic jump in diverging channels

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ABSTRACT

This paper addresses the integral conservation of linear and angular momentum in the steady hydraulic jump in a linearly diverging channel.

The flow is considered to be divided into a mainstream that conveys the total liquid discharge, and a roller where no average mass transport occurs. It is assumed that no macroscopic rheological relationship holds, so mass, momentum and angular momentum integral balances are independent relationships. Normal stresses are assumed to be hydrostatic on vertical, cylindrical surfaces. Viscous stresses are assumed to be negligible with respect to turbulent stresses. Assuming that the horizontal velocity distribution in the mainstream is uniform and that the horizontal momentum and angular momentum in the roller are negligible with respect to their mainstream counterparts, an analytical solution is obtained for the free surface profile of the flow. This solution is fundamental for finding the sequent depths and their positions. Consequently, it permits solving for the length of the jump, which is assumed to be equal to the length of the roller. Mainstream and roller thicknesses can also be derived from the present solution. This model may also be theoretically used to derive the average shear stresses exerted by the roller on the mainstream and the power losses per unit weight. This second relationship, which returns the well-known classical expression for total power loss in the jump, demonstrates that the strongly idealized mechanical model proposed here is internally consistent.

1. Introduction

The importance of the hydraulic jump for environmental hydraulics and hydraulic engineering is well known. This phenomenon is highly relevant in both fluid mechanics and classical hydraulics. From the point of view of mathematical analysis of shallow water equations, a hydraulic jump represents a singularity that breaks the solution continuity and completely changes the role of the boundary conditions. If a jump occurs, both upstream and downstream boundary conditions influence water elevation [1].

This work addresses a classical problem in applied hydraulics. It is inspired by the work of the first author [2], who identified the problem of unbalanced angular momentum in the hydraulic jump for wide rectangular channels. In several applications, a diverging channel is used to force the jump in shorter lengths and to increase control over the position of the jump.

Many works in the available literature concern small-scale problems that are dominated by viscosity and surface tension (see, for example [3], and the reference therein). In contrast, this paper focuses on fully developed turbulent flow over a smooth bottom with negligible surface tension and viscosity effects. The studied case has important applications for large-scale hydraulic engineering because it pertains to the radial flow in stilling basins more than the small jump created in the laboratory.

Basic theoretical and experimental analyses of the hydraulic jump are based on the fundamental studies by Bakhmeteff and Matzke [4] and Rouse et al. [5]. Recent work has incorporated modern flow visualization and laser Doppler anemometry techniques [6–8]. A more extensive review can be found in [2].

From a large-scale point of view, it is worth noting that a huge amount of scientific literature has been recently published concerning one- or two-dimensional numerical schemes for simulating steady jump and moving bore propagation (i.e., [9–11]). Nevertheless, it is important to recall that all of these schemes necessarily treated a bore as a step with vanishing length. In fact, the less a numerical method introduces diffusion effects, the more it can be considered a good, high-resolution method.

The present approach provides unique physical insights on the phenomenon and its spatial development despite the fact that an exhaustive treatment of turbulence is completely omitted. This work follows the typical approach of the hydraulic engineering community, in which turbulence effects are averaged over both a

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Nomenclature

$\mathbf{e}_r$	unit vector, r direction
$\mathbf{e}_z$	unit vector, z direction
$\mathbf{v}$	velocity vector [m/s]
a_χ	non-dimensional constant in Eq. (49) [0]
b	typical distance from the origin of the action line of vertical forces [m]
g	gravity acceleration [m^2/s]
p	pressure [Pa]
p_{RS}	pressure exerted on the mainstream by the roller [Pa]
q	generic distributed quantity, see Fig. 1
r	radial coordinate [m]
r_c	critical radius [m]
s	streamline dividing the mainstream from the roller [m]
t	dummy variable in Eq. (85) [0]
v_r	radial velocity [m/s]
v_z	vertical velocity [m/s]
v_θ	tangential velocity [m/s]
x	axial coordinate [m]
y	non-dimensional depth [0]
y_1	upstream non-dimensional depth [0]
y_2	downstream non-dimensional depth [0]
y_{10}	guess value of upstream non-dimensional depth [0]
y_{20}	guess value of downstream non-dimensional depth [0]
z	vertical coordinate [m]
A_M	moment of longitudinal momentum crossing the whole depth [N m]
A_{M_R}	moment of longitudinal momentum crossing the roller [N m]
A_{M_S}	moment of longitudinal momentum crossing the mainstream [N m]
A_{M_z}	moment of vertical momentum crossing the whole stream [N m]
$A_{M_{Rz}}$	moment of vertical momentum crossing the roller [N m]
$A_{M_{Sz}}$	moment of vertical momentum crossing the mainstream [N m]
$A_{F_{RSx}}$	moment of the horizontal forces acting on the mainstream by the roller [N m]
$A_{F_{RSz}}$	moment of the vertical forces acting on the mainstream by the roller [N m]
A_T	moment of vertical tangential stress on the whole stream [N m]
A_{T_R}	moment of vertical tangential stress on the roller [N m]
A_{T_S}	moment of vertical tangential stress on the mainstream [N m]
A_Π	moment on the whole stream due to the pressure force [N m]
A_{Π_L}	moment on the whole stream due to the pressure force by the lateral walls [N m]
$A_{\Pi_{LR}}$	moment on the roller due to the pressure force by the lateral walls [N m]
$A_{\Pi_{LS}}$	moment on the mainstream due to the pressure force by the lateral walls [N m]
A_{Π_R}	moment on the roller due to the pressure force [N m]
A_{Π_S}	moment on the mainstream due to the pressure force [N m]
C	constant in Eq. (73)
C_1	non-dimensional constant in Eq. (83) [0]
E	specific energy of the flow [m]
E_1	upstream specific energy [m]
E_2	downstream specific energy [m]
Fr	Froude number of the flow [0]
Fr_1	upstream Froude number [0]
Fr_{20}	guess value of downstream Froude number [0]
$\mathcal{F}_1$	non-dimensional total force of the flow at the upstream jump position [0]
$\mathcal{F}_s$	non-dimensional static force in a generic position [0]
F_{RSx}	horizontal force acting on the mainstream from the roller [N]
F_{RSz}	vertical force acting on the mainstream from the roller [N]
$\mathcal{F}_L$	non-dimensional force exerted by the lateral walls [0]
$\mathcal{F}_t$	non-dimensional total force through a section [0]
H	total(=piezometric + kinetic) head of the flow [m]
$\mathcal{I}$	non dimensional total force due to the lateral walls [0]
M_R	roller momentum flux [N]
M_S	mainstream momentum flux [N]
M_{Rz}	vertical momentum crossing the roller [N]
M_{Sz}	vertical momentum crossing the mainstream [N]
M_z	vertical momentum crossing the whole stream [N]
O	pole for angular momenta
U	cross section averaged velocity [m/s]
Q	liquid discharge [m^3/s]
$\mathcal{Q}$	generic integral quantity, see Fig. 1
R	(vertical) reaction force on the mainstream from the bottom [N]
T	(vertical) total tangential forces on vertical sections [N]
T_R	(vertical) total tangential forces on vertical sections of the roller [N]
T_S	(vertical) total tangential forces on vertical sections of the mainstream [N]
W	(vertical) mass force on the whole depth [N]
W_R	(vertical) mass force on the roller [N]
W_S	(vertical) mass force on the mainstream [N]
Y	current depth [m]
Y_1	upstream depth of the jump [m]
Y_2	downstream depth of the jump [m]
Y_c	critical depth [m]
V	volume of the whole stream inside the jump [m^3]
V_R	volume of the roller inside the jump [m^3]
V_S	volume of the mainstream inside the jump [m^3]
$\mathcal{V}$	non-dimensional volume of the whole stream inside the jump [0]
$\mathcal{V}_R$	non-dimensional volume of the roller inside the jump [0]
$\mathcal{V}_S$	non-dimensional volume of the mainstream inside the jump [0]
α	half angular amplitude of the channel [0]
α_1	parameter in Eqs. (A.7) and (A.8) [0]
α_2	parameter in Eqs. (A.7) and (A.8) [0]
$\tan(\beta)$	jump local steepness [0]
$\tan(\beta_1)$	jump local steepness at the upstream section of the jump [0]
β_s	momentum coefficient for the mainstream flow [0]
γ	specific weight of the fluid [N/m^3]
χ	non-dimensional constant in Eq. (19) [0]
δ	roller thickness [m]
η	mainstream thickness [m]
ρ	density of the fluid [kg/m^3]
σ	non dimensional roller thickness [0]
$\bar{\tau}$	vertically averaged tangential stress on vertical sections [Pa]
τ_{rz}	z-component of the local turbulent stress on a cylindrical surface normal to $\mathbf{e}_r$ [Pa]
τ_{RS}	total tangential stress exerted on the mainstream by the roller [Pa]
$\bar{\tau}_R$	vertically averaged tangential stress on the roller [Pa]
$\bar{\tau}_S$	vertically averaged tangential stress on the mainstream [Pa]
θ	angular coordinate [0]
ξ	non-dimensional radius [0]