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Saltwater intrusion modeling in heterogeneous confined aquifers using two-relaxation-time lattice Boltzmann method

Borja Servan-Camas [a,1], Frank T.-C. Tsai [b,*]

[a] International Center for Numerical Methods in Engineering, Universidad Politecnica de Cataluña, Gran Capitan s/n, 08034 Barcelona, Spain
[b] Department of Civil and Environmental Engineering, Louisiana State University, 3148G Patrick F. Taylor Hall, Baton Rouge, LA 70803-6405, United States

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ABSTRACT

This study develops a lattice Boltzmann method (LBM) with a two-relaxation-time collision operator (LTRT) to solve saltwater intrusion problems. A directional-speed-of-sound (DSS) technique is introduced to take into account the hydraulic conductivity heterogeneity and discontinuity, as well as the velocity-dependent dispersion coefficient. The forcing terms in the LTRT model are customized in order to recover the density-dependent groundwater flow and mass transport equations. Using the LTRT with the squared DSS achieves at least second-order accuracy. The LTRT results are verified with Henry's analytical solution as well as compared with several numerical examples and modified Henry problems that consider heterogeneous hydraulic conductivity and velocity-dependent dispersion. The numerical results show good agreement with the Henry analytical solution and with the numerical solutions obtained by other numerical methods.

1. Introduction

Saltwater intrusion in aquifers is often described by coupled density-dependent groundwater flow and advection–dispersion equations because of hydrodynamic dispersion and a wide transition zone [1–5]. Numerical methods for solving the saltwater intrusion problem include the method of characteristics (MOC) [6], the finite element method (FEM) [7] and the finite difference method (FDM) [8,9]. Recently, the mixed hybrid finite element (MHFE) and discontinuous finite element (DFE) methods were developed to increase numerical stability when solving variable density flow and transport problems [10,11]. To authors' knowledge, there is very little discussion on the implementation of the lattice Boltzmann method (LBM) to saltwater intrusion problems in aquifers.

In the past years, the LBM has received increasing attention on transport problems because the LBM solves macroscopic equations based on microscopic models and mesoscopic kinetic equations. Although initially developed for solving hydrodynamic problems [12], the LBM has been developed to solve various transport problems such as the reaction–diffusion equation [13], the contaminant transport equation [14], the coupled flow and heat/mass transfer problem due to density dependency [15], the density-dependent flow problem in porous media [16] and the anisotropic advection–dispersion equation (AADE) [17,18]. Although in Zhang et al. [17] the anisotropic dispersion tensor was recovered by using direction-dependent relaxation times, solving the AADE using the LBM remains challenging when the principal directions of dispersion anisotropy are not aligned with the lattice directions and the ratio of longitudinal to transverse dispersivities is high. Moreover, as an extension of Ginzburg [18], the LBM has been also applied to solving groundwater flows in heterogeneous stratified aquifers in Ginzburg [19] and in Ginzburg and d'Humieres [20] by maintaining the continuity of the normal component of the Darcy velocity (total flux) at the interface between layers. These layers were horizontal and aligned with the lattice orientations, which may present a limitation to the general case. In this work, we develop a technique to handle heterogeneous and discontinuous hydraulic conductivity with arbitrary lattice orientations.

The fast development of LBM arises from: first, the LBM is easy to implement because it deals with complex equations by solving explicitly a system of first-order linear differential equations (the Boltzmann equation); second, the particle-based description of the LBM provides a new way to implement complex boundary conditions, e.g., the mass flux boundary condition [21,22]; third, the

* Corresponding author. Tel.: +1 225 578 4246; fax: +1 225 578 4945.
E-mail addresses: bservan@cimne.upc.edu (B. Servan-Camas), ftsai@lsu.edu (F.T.-C. Tsai).
[1] Formerly, graduate student of Department of Civil and Environmental Engineering, Louisiana State University.

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LBM is an explicit scheme that does not involve the resolution of any global system of equations; and fourth, only information from neighboring nodes (locality) is needed for evolving variables. Therefore, the explicit nature along with the locality property makes the LBM ideal for parallel computing.

The purpose of this study is to improve the LBM by using a two-relaxation-time (TRT) collision operator and a directional-speed-of-sound (DSS) technique to cope with heterogeneous hydraulic conductivity and hydrodynamic dispersion in the saltwater intrusion problem. The lattice Boltzmann equation with the TRT collision operator is known as the LTRT model [18]. The LTRT will provide additional flexibility to improve the accuracy. We will introduce the DSS to take into account the hydraulic conductivity heterogeneity and the velocity-dependent dispersion coefficient. The analytical solutions of the original and modified Henry problems will be used to verify the LBM results. The LBM solutions will also be compared with other numerical methods.

2. Density-dependent saltwater intrusion model

Darcy's law for groundwater flow is [23]

$$\mathbf{q} = -\frac{k}{\mu}(\nabla p - \rho \mathbf{g}) \tag{1}$$

where $\mathbf{q}$ is the Darcy velocity or Darcy flux, k is the intrinsic permeability, μ is the dynamic viscosity of water, ρ is the density of water and is a function of space and time, p is the pore water pressure, $\mathbf{g}$ is the vector of gravitational acceleration and ∇ is the gradient operator. The hydraulic conductivity is defined as $K = k\rho g/\mu$. This study focuses on saltwater intrusion simulation in a two-dimensional confined aquifer in a vertical plane, where the maximum salinity considered is the same as that of seawater. The density-dependent groundwater flow equation in terms of the fresh groundwater head is [24–26]

$$\phi S_{s_f}\frac{\partial h_f}{\partial t} + \frac{n}{\rho_f}\frac{\partial \rho}{\partial C}\frac{\partial C}{\partial t} = \left[\frac{\partial}{\partial x}\left(\phi K_f \frac{\partial h_f}{\partial x}\right) + \frac{\partial}{\partial z}\left(\phi K_f \frac{\partial h_f}{\partial z}\right) + \frac{\partial}{\partial z}(\phi(\phi-1)K_f)\right] + \frac{\rho_{ss}}{\rho_f}Q_{ss} \tag{2}$$

where S_{s_f} is the freshwater specific storage, C is the salt concentration, $h_f = p/\rho_f g + z$ is the fresh groundwater head, $K_f = k\rho_f g/\mu$ is the freshwater hydraulic conductivity, n is the porosity, ρ_f is the freshwater density, $\phi = \rho/\rho_f$ is the density ratio, ρ_{ss} is the water density at the sinks/sources, Q_{ss} is the volumetric flow rate per unit aquifer volume at the sinks/sources, x is the spatial coordinate in the horizontal direction, z is the spatial coordinate in the upward vertical direction and t is the time variable.

A linear relationship between the water density ratio and salt concentration is observed for salt concentrations less than 35 parts per thousand (ppt) [27]. Therefore, the density ratio is considered to be $\phi = 1 + EC$ in this study, where E is a constant.

The salt transport equation in porous media is given by the advection–dispersion equation (ADE):

$$\frac{\partial (nC)}{\partial t} + \nabla \cdot (n\mathbf{u}C) = \nabla \cdot (nD\nabla C) + C_{ss}Q_{ss} \tag{3}$$

where $\mathbf{u} = \mathbf{q}/n$ is the average pore velocity, D is the dispersion coefficient and C_{ss} is the salt concentration at the sinks/sources. The porosity is considered not uniform $n = n(\mathbf{x})$. This study focuses on the isotropic advection–dispersion equation (IADE), where the hydrodynamic dispersion is isotropic and velocity-dependent [5]:

$$D = \kappa|\mathbf{u}| + D_m \tag{4}$$

where κ is the dispersivity and D_m is the molecular diffusion coefficient.

3. LBM for transport equation

3.1. Lattice Boltzmann method with two-relaxation-time collision operator (LTRT)

This study develops a lattice Boltzmann method with the two-relaxation-time collision operator (LTRT) [18,28] for saltwater intrusion simulation in confined aquifers. The discrete velocity fields considered in this study include a D2Q5 lattice (two dimensions and five discrete velocities) and a D2Q9 lattice (two dimensions and nine discrete velocities), shown in Fig. 1, which have zero velocity for resting particles. The LTRT is

$$f_i(\mathbf{x} + \mathbf{c}_i\Delta t, t + \Delta t) = f_i(\mathbf{x}, t) - \frac{1}{\tau_s}(f_i^s(\mathbf{x}, t) - f_i^{seq}(\mathbf{x}, t)) - \frac{1}{\tau_a}(f_i^a(\mathbf{x}, t) - f_i^{aeq}(\mathbf{x}, t)) + \Delta t F_i \tag{5}$$

where $f_i(\mathbf{x} + \mathbf{c}_i\Delta t, t + \Delta t)$ represents the post-collision particle distribution function streaming along direction i to location $(\mathbf{x} + \mathbf{c}_i\Delta t)$ at time $t + \Delta t$; f_i^s and f_i^{seq} are the symmetric parts of the pre-collision particle distribution function and the equilibrium distribution function (EDF), respectively; f_i^a and f_i^{aeq} are the anti-symmetric parts of the pre-collision particle distribution function and the EDF, respectively; τ_s and τ_a are the symmetric relaxation-time and anti-symmetric relaxation-time, respectively; F_i is the forcing term along i direction; $\mathbf{c}_i$ is the particle velocity along the i direction; and Δt is the time step. Eq. (5) reduces to the lattice Boltzmann method with the Bhatnagar–Gross–Krook (BGK) collision operator [29], known as LBGK, when $\tau_s = \tau_a$.

In the two-relaxation-time collision operator, particle distribution functions are relaxed to the equilibrium state by relaxing their symmetric and anti-symmetric parts separately, which are given by

$$f_i^s = \frac{f_i + \bar{f}_i}{2}; \quad f_i^a = \frac{f_i - \bar{f}_i}{2}$$
$$f_i^{seq} = \frac{f_i^{eq} + \overline{f_i^{eq}}}{2}; \quad f_i^{aeq} = \frac{f_i^{eq} - \overline{f_i^{eq}}}{2} \tag{6}$$

where $\bar{f}_i$ and $\overline{f_i^{eq}}$ are the particle distribution function and equilibrium distribution function (EDF) along the opposite direction of i. The first four moments of the EDFs are

$$\sum_i f_i^{eq} = \sum_i f_i^{seq} = M_0; \quad \sum_i f_i^{aeq} = 0 \tag{7}$$

$$\sum_i f_i^{eq} c_{i\alpha} = \sum_i f_i^{aeq} c_{i\alpha} = M_{1\alpha}; \quad \sum_i f_i^{seq} c_{i\alpha} = 0 \tag{8}$$

$$\sum_i f_i^{eq} c_{i\alpha}c_{i\beta} = \sum_i f_i^{seq} c_{i\alpha}c_{i\beta} = M_{2\alpha\beta}; \quad \sum_i f_i^{aeq} c_{i\alpha}c_{i\beta} = 0 \tag{9}$$

$$\sum_i f_i^{eq} c_{i\alpha}c_{i\beta}c_{i\gamma} = \sum_i f_i^{aeq} c_{i\alpha}c_{i\beta}c_{i\gamma} = M_{3\alpha\beta\gamma}; \quad \sum_i f_i^{seq} c_{i\alpha}c_{i\beta}c_{i\gamma} = 0 \tag{10}$$

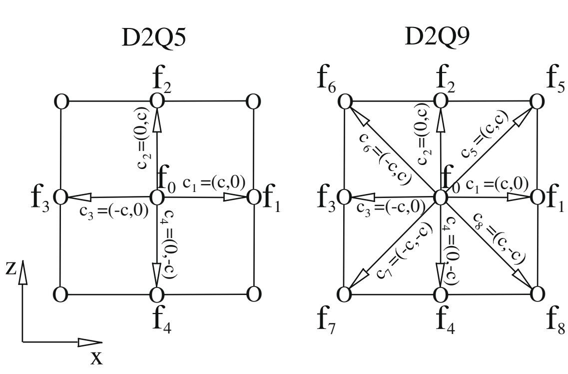

Fig. 1. D2Q5 and D2Q9 lattices.