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Hyperslow drainage in a porous medium Influence of retention curve

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ABSTRACT

We investigate the process of very slow drainage from pore network simulations in relation with the study of the repository of radioactive waste materials in deep geological formation. The process is characterized by a quite low desaturation of the medium. Considering the special case of a constant stabilizing gradient, we study the sensitivity of the solution given by the classical continuum two-phase flow model to the shape of the retention curve. The solution is found to be very sensitive to the evolution of the retention curve in the range of the high wetting fluid saturations. We show how to construct the retention curve so as to minimize the prediction error with the classical two-phase flow model.

1. Introduction

Drainage in a porous medium, i.e. the immiscible displacement of a wetting fluid by a non-wetting fluid, has been the subject of many studies, notably in relation with petroleum engineering or hydrology applications. Simulations of drainage for a particular application are usually performed using a classical phenomenological model, referred to as the classical two-phase flow model (CTPFM) in this article. This model is based on the well-known concepts of capillary pressure curve, relative permeabilities and generalized Darcy's law, e.g. [1]. This model has been widely used and is very often reasonably successful in predicting drainage. It is the obvious tool for simulating the drainage process from an engineering perspective. Yet, there are situations where this model necessarily leads to poor predictions. For example, with the ultrathin porous layers encountered in the proton exchange membrane fuel cells (PEMFCs). There is a lack of length scale separation, some layers are typically 4 to 10 pore size wide only, and the regime is dominated by capillary effects; two reasons that lead to poor predictions with the CTPFM, e.g. [2]. There are also situations where the lack of length scale separation cannot be invoked but where the ability of the CTPFM model to predict the drainage displacement is however unclear and must be studied.

The example motivating the present work has to do with the study of the repository of radioactive waste materials in deep geological formations. In a radioactive waste repository, several processes lead to the formation of gas. One of the most important is corrosion of ferrous materials under anoxic conditions, a process leading to the production of a large amount of hydrogen. The perturbation induced by gas in such repository has to be studied and especially its consequences on the host rock, which is a low permeability rock ($k \sim 10^{-20}$ m^2). It is anticipated that the gas production will exceed the host rock capacity to eliminate the gas in pore water by dissolution process only. Gas will continue to accumulate until its pressure becomes sufficiently large to enter the host rock, causing therefore the displacement of the water saturating the pore space at the beginning of the displacement. Hence, a rather classical capillary-viscous drainage is expected to occur in the host rock owing to the hydrogen pressurisation in the repository, at least as long as the gas pressure is not too high for not inducing micro- or macro-mechanical disorders, see [3] for more detail. As will be discussed in more detail below, the displacement is, however, extremely slow because of the low hydrogen production rate in the system. Note, however, that this production is supposed to occur over about 100 000 years. Hence a very low production rate over such a long period eventually leads to a non-negligible volume of gas. As a result of this very low production rate, the simulations based on the CTPFM lead to very small desaturations of the host rock, typically in the range [0.95–1] of liquid water saturation, e.g. [4]. This range of saturations typically correspond to a range where the experimental determination of the capillary pressure curve as well as the gas relative permeability are extremely delicate, especially for low permeability rocks. In fact, data are very often not available in this range of saturations and the behaviour of the parameters in this range of saturations has therefore to be extrapolated in a way or another from the data available for lower saturations. Hence a first problem is that the accurate evolution of these key parameters of the CTPFM is generally not known in the range of saturations predicted with the

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CTPFM. One question is therefore to evaluate the sensitivity of the model to these parameters for this range of saturation.

There is more, however, as can be discussed in relation with the theory of drainage based on the representation of a porous medium as a lattice of interconnected pores and invasion percolation concepts, e.g. [5–8]. It is well established that invasion percolation (IP) describes very well the drainage on a pore network for the asymptotic case where the displacement is controlled by the capillary forces only, e.g. [5]. When viscous effects are non-negligible, drainage can be analyzed using the concept of invasion percolation in a gradient (IPG), at least for a sufficiently slow displacement. Applying this theory leads to the conclusion that the drainage displacement involved in the nuclear waste repository problem is typically an IP/IPSG transition flow, where IPSG stands for Invasion Percolation in a Stabilizing Gradient (details are given in Section 2). Whereas the CTPFM can well predict a well developed IPSG drainage displacement (except for a very small region ahead of the invasion front), the question is quite unclear for an IP/IPSG displacement. To gain insights into this problem, we consider a simpler IP/IPSG transition flow, namely a displacement in a constant gradient. This corresponds for example to a very slow drainage in the presence of stabilizing gravity forces. The advantage of the simpler situation is twofold: i) a simple analytic solution can be obtained from the CTPFM, ii) the pore network simulations that are used to generate the IP/IPSG solution are significantly simpler and much less computation time consuming than when viscous effects are responsible for the percolation gradient. The drawback is that only the sensitivity to the retention curve is addressed. To assess the performances of the CTPFM for this situation, comparisons are made with pore network simulations. Pore network simulations are also used to determine the capillary pressure–saturation relationship to be used with the CTPFM using the same pore network structure as in the "direct" IP/IPSG simulations.

Since we consider a somewhat simpler situation, the present study should be considered as a first step in the assessment of the CTPFM for the repository problem. Naturally, it might be objected that the host rock, the Callovo–Oxfordian argillite located in Eastern France, is a very complex clay rock difficult to represent *a priori* in terms of a simple ordinary pore network. There is no such claim in the present work. The idea is simply to consider a model problem of broader interest allowing us to shed light on a generic problem: the ability of the CTPFM to simulate an IP/IPSG transitional flow. Naturally, we believe that the findings of the study are of direct interest to the repository problem. The possibility of representing the host rock with a PNM requires, however, a specific study and this is left for a future work. Also, we neglect the gas dissolution process in the analysis. Hydrogen partitioning and subsequent advection–diffusion transport of dissolved hydrogen in the liquid is actually expected to occur in the repository problem. One interesting question beyond the scope of the present work is to study the possible modifications that the consideration of this process might induce in the approach presented here. This is also left for a future work.

The paper is organised as follows: first we apply in Section 2 the IP/IPSG drainage theory to our particular slow displacement problem. The travelling front solution given by the CTPFM is derived in Section 3. The pore network model is presented in Section 4 and the algorithm used to determine the retention curve from pore network simulations in Section 5. Comparisons between the continuum model solution and the pore network simulations for a constant percolation gradient are presented in Section 6. Implications of the results are discussed in Section 7.

2. Drainage theory

The different asymptotic patterns that are expected when capillary or viscous forces control the drainage process are well summarized in the phase diagram proposed by Lenormand [5,9]. This diagram is schematically shown in Fig. 1. The displacement is characterized by two dimensionless numbers: the capillary number $\mathrm{Ca} = U\mu_{nw}/\gamma$, where U is the injection velocity, μ_{nw} the viscosity of the displacing (non-wetting) phase and γ the interfacial tension between the two fluids; the viscosity ratio $M = \mu_w/\mu_{nw}$, where μ_w is the viscosity of the displaced phase. Hence the first step is to estimate these numbers for our problem. The production of H_2 is estimated at about 50 mol/year per unit length of gallery. The lateral area, through which the gas can invade the host rock, of such a section is about 80 m^2. This leads to a filtration velocity U of about 4×10^{-10} m/s and for hydrogen and water to $\mathrm{Ca}\approx10^{-14}$ and $M = 10^2$. Hence the displacement is at extremely low capillary number. A crude application of Lenormand's phase diagram would lead to the conclusion that the displacement is in the invasion percolation (IP) regime, that is a regime controlled by capillary effects only. It is well known that this regime is not compatible with the CTPFM owing to the fractal structure of the capillary fingering pattern corresponding to IP, e.g. [2] for an example of comparison between the CTPFM and pore network simulations (PNS) in this regime. As pointed out in [7] or [9], however, the length of the displacement has to be taken into account in the analysis. For a given (small) capillary number, the IP regime prevails as long the length of the system or the length of the zone visited in the displacement (in an unbounded domain) is not too large. For greater lengths, the pressure gradients due to viscous effects do influence the displacement and the pattern is no longer an IP pattern.

As analyzed in details in [6,7], two regimes can then occur when the effect of viscous forces become significant: a stabilized displacement or capillary-viscous fingering. The first one is analyzed as an invasion percolation process in a stabilizing gradient (IPSG) whereas the second one is analyzed as an invasion percolation process in a destabilizing gradient (IPDG). The theory developed in [6,7] provides a criterion to delineate the two regimes. Assuming for simplicity as in [6,7] drainage in a rectilinear porous medium, the displacement is expected to be stabilized when

$$\frac{Ca}{\Sigma} M^{[\zeta + 1 + \nu(D-1)]/[\zeta + \nu(D-2)]} \ll 1 \qquad (1)$$

where Σ is the dimensionless standard deviation of the pore throat size distribution, ζ the conductance exponent of percolation theory,

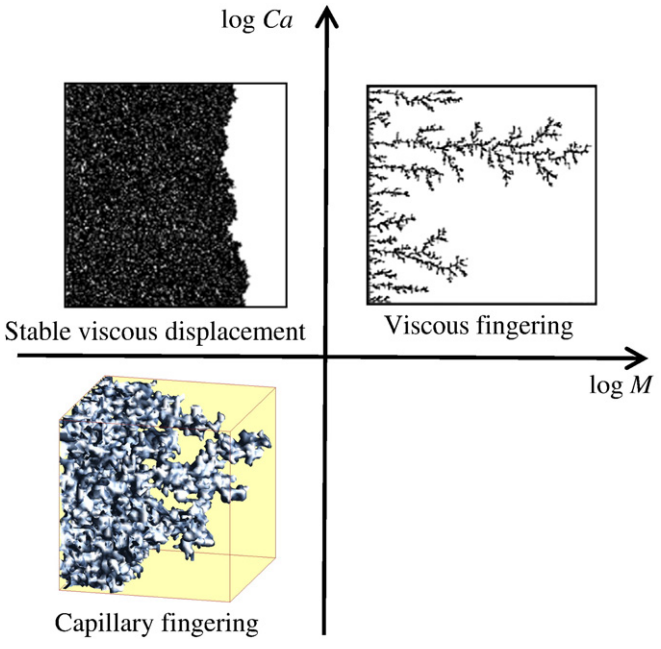

Fig. 1. Drainage phase diagram. Adapted from [9]).